SYMBOLISM AND TRUTH

CHARLES SANDERS PEIRCE

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SYMBOLISM AND TRUTH

LONDON : HUMPHREY MILFORD

OXFORD UNIVERSITY PRESS

SYMBOLISM AND TRUTH

AN INTRODUCTION TO THE THEORY OF KNOWLEDGE

BY

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CAMBRIDGE, MASS., U. S. A.

TO

H. B. E.

IN GRATITUDE FOR
FAITH AND WORKS

UNITED THEOLOGICAL SEMINARY
AR 2 3 1959

When I heard the learn'd astronomer;
When the proofs, the figures, were ranged in columns before me;
When I was shown the charts and the diagrams, to add, divide, and measure them;
When I, sitting, heard the astronomer, where he lectured with much applause in the lecture-room,
How soon, unaccountable, I became tired and sick;
Till rising and gliding out, I wander'd off by myself,
In the mystical moist night-air, and from time to time,
Look'd up in perfect silence at the stars.

WALT WHITMAN, *Drum Taps, 1865*

PREFACE

THE theory of knowledge, occupying as it does the borderland between psychology, logic, and metaphysics, is a peculiarly difficult subject to isolate and study in itself. The materials are widely scattered through philosophical literature, discussions of the problems appear in works of the most diverse character, and any one who attempts to single out the essential questions will be sure to omit some that are important in the eyes of many people and to include others that might be omitted. He will place his emphasis somewhere, with the result that he will fail to stress points that perhaps equally deserve emphasis. By way of remedy therefore he ought to indicate his angle of approach and call attention to what he believes are the major gaps in his treatment.

The method of the present work is mainly critical and analytic, rather than speculative.[1] A single line of attack, which goes at once to the heart of the problem, is chosen—namely, the rôle of symbols in knowledge; and about this the entire analysis is organized. Knowledge is inseparable from its expressions; a study of these expressions should therefore throw light on the theory of knowledge as a whole. With this conviction, I begin by examining *meaning*. But unfortunately the psychology of meaning is still in a fluid state, and the most that one who is not a psychologist can do is to point out in a rough way that to which, in his own experience, he gives the name "meaning." The consideration of the logical forms of meanings leads to a discussion of the nature of facts, relations, qualities, universals and indi-

[1] See Mr. C. D. Broad's interesting statement of the difference between critical and speculative philosophy in *Contemporary British Philosophy*, edited by Professor J. H. Muirhead (1924), pp. 63 ff.; also, Broad's *Scientific Thought*, (1923), introduction.

viduals, classes, description, synthesis and analysis, possibility, and finally to the definition and tests of truth. The question of truth is naturally linked to that of belief, judgment, and negation; and since logical form stands out most clearly in quasi-mathematical deductive systems, a chapter is given to the study of these systems.

Metaphysical ideas are kept as far as possible in the background. Speculations concerning the relation of knowledge to an ultimate reality, that is, the issues of idealism and realism, of gnosticism and agnosticism, of monism and pluralism, of the final validity of intuition as opposed to reason, are postponed until the last chapter; for a theory of the relation of knowledge to reality can be successfully held only after the ground has been cleared by an analysis of knowledge as a phenomenon. By examining in detail the elements that make up the complex process, knowledge, I hope to introduce the reader to the wider speculative aspects of the subject; but in this direction the line of further thought is merely sketched.

Discussions of the theory of knowledge often begin by classifying different types of views and proceed to compare these in the effort to distill some truth from each. This method is useful; it sets in order, for one unfamiliar with philosophy, a large amount of material; but it tends to become a study of conflicting schools of opinion on the problem of knowledge rather than of the problem itself. No systematic classification of epistemological theories is given here, though an acquaintance with such classifications, and with the history of philosophy, will be helpful to the reader.

The central place of the concept of logical form gives to certain parts of the book a superficial resemblance to Mr. L. Wittgenstein's recent *Tractatus Logico-Philosophicus*. How deep this resemblance goes I am unable to say, but the language is similar. "We make to ourselves (in thought) pictures of facts," says Mr.

Wittgenstein. "In the picture and the pictured there must be something identical in order that the one can be a picture of the other at all. What the picture must have in common with the reality in order to be able to represent it after its manner — rightly or falsely — is its form of representation." And Mr. Bertrand Russell adds to this the comment, "We speak of a logical picture of a reality when we wish to imply only so much resemblance as is essential to its being a picture in any sense, that is to say, when we wish to imply no more than an identity of logical form." [1]

The positive or descriptive theory of knowledge of the first seven chapters, which attempts cautiously to thread its way between psychology and metaphysics, is no doubt allied to the type of philosophy known in Germany as *phänomenologie*. But it leans much more heavily towards psychology than does the *phänomenologie* of E. Husserl's *Logische Untersuchungen*. Similarly, A. Meinong's theory of the *objective*, set forth in his *Untersuchungen zur Gegenstandstheorie und Psychologie*, in his studies of Hume and other works, appears to me to remove meaning too completely from its psychological setting and to view as simple what is in reality complex. However, I am at one with the phenomenological school in wishing to treat the theory of knowledge as an autonomous subject, that is, in desiring to assure it as much autonomy as belongs to any other branch of philosophy or to psychology. It is clear that knowledge cannot be wholly described in psychological terms. Hume attempted this and failed. But it is clear also that nothing is gained by trying to banish from knowledge all concrete psychological factors.

Among the topics that might be included in such a critical study of knowledge but that are omitted or only lightly touched here, are the following: the knowledge of other minds and of the

[1] L. Wittgenstein, *Tractatus Logico-Philosophicus* (1922), introduction, p. 10.

self; the knowledge of values and the relations of theories of truth to theories of moral and aesthetic value; the validity of inductive inferences; the nature of scientific hypotheses, especially the place of elegance, economy, and completeness in scientific theories; the knowledge of space and time. (The allusions to space and time made in connection with the discussion of the individual in Chapter III need to be supplemented by a detailed treatment, such as that of Mr. A. N. Whitehead's *The Principles of Natural Knowledge* and Mr. C. D. Broad's *Scientific Thought*. Particularly is this so at the present time, in view of the theory of relativity.) But what is said here of the fundamental factors in knowledge may suggest ways of approach to these more tangled problems.

For the use of the student, a list of selected readings bearing on the subjects discussed in each chapter is given. This makes available for study along with the text some of the widely scattered literature of the theory of knowledge, and will aid the student in obtaining a complete and systematic grasp of this branch of philosophy.

R. M. E.

CAMBRIDGE, MASS.
January, 1925.

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SYMBOLISM AND TRUTH

INTRODUCTION

I

As the various sciences have branched off from the common stem of philosophy, which was in the beginning the pursuit of truth wherever it might be found, the philosophers have been left with certain problems preëminently their own, and among these is the theory of knowledge. It is not probable that the theory of knowledge will ever be a science in the sense in which mathematics, physics, and biology are sciences. There is too much room for difference of opinion, not only in the solution of its problems, but in their statement. Yet progress in this field of thought, as in any other, comes only with the attempt to define the subject-matter more clearly, to cut it off from neighboring fields which threaten it with incursions of irrelevant questions, and to find some ideas that bind the whole together.

Just as a physical theory takes its rise from physical facts, so a theory of knowledge is built on the facts of knowledge; but though these facts are before us when we make the simplest statement or perform the most elementary process of reasoning, they are by no means so apt to strike the mind as are the facts of physics or of any other special science. We are mentally far-sighted and tend to neglect that which is closest to us.

Everything that can be mentioned is *known* in some sense. If I deny the existence of two-headed lions or red elephants, I know what I mean by these denials, and therefore these imaginary creatures are somehow objects of my knowledge. If I open my eyes on the world about me, I see trees and houses, and people passing in the street; and these are obviously known in some other way than the red elephants and the two-headed lions. The geometer sketches a rough triangle on a bit of note-

paper, and on this diagram demonstrates that the sum of the three interior angles of the triangle is equal to a straight angle. This exact geometrical proposition is not true of the imperfect penciled triangle; it is true of an ideal Platonic triangle, which is only suggested by the rough figure and is apprehended through the eye of reason rather than the eye of sense. The ideal triangle belongs thus to another class of objects of knowledge, just as do the ideal gas, the frictionless motion, the perfectly rigid and elastic body, of the physicists. Again, what is one to say of dream objects? The events and people of dreams are for the time as intensely real as the events and people in the street outside at the present moment. What is the reason for our prejudice in favor of the reality of the things of waking life, which fill at best only three-quarters of our experience? Certainly the dream is a form of knowledge.

There are evidently many different senses of the term "knowledge." We dwell in the medium of the known, which surrounds us as an atmosphere. This very ubiquity makes it difficult to stand off from knowledge and analyze it. Perhaps the simplest way of marking out the subject-matter of the theory of knowledge is this: Anything of which it can be said that it is *known* in any sense is a proper subject of our investigation. The theory of knowledge is interested in these things not as physical, psychological, mathematical, or any other particular sort of objects, but as *things known*. Although mathematical equations, physical, chemical, and biological laws, historical facts, and a multitude of other data which belong to the special sciences are also data of the theory of knowledge, this theory looks on them in a different light from any of the special sciences. What common factors, laws, uniformities, relations, belong to things merely as objects of knowledge? This is the question that must be asked.

Thus the theory comprehends in its subject-matter all the special sciences. These are particular cases of knowledge from

which the general principles are to be induced. But it attacks problems that none of these sciences can meet. With Bacon it "takes all knowledge to be its province" and views this province from an angle of its own. It includes much of what is often classified as philosophy of science or logic, for it considers such questions as what is a proposition, what is a scientific law, what are truth, error, assertion, belief, conception, meaning? In short, wherever there are principles of the known *qua* known, these principles are a part of the theory of knowledge.

Meaning, for example, is present in all knowledge, from the simplest perception to the most complex mathematical expression. Any knowledge that does not make use of meaning is an immediate awareness, an intuition, which can give no rational account of itself. Just what *meaning* is is a thorny question. But if a satisfactory answer can be given, the description of knowledge will have been pushed forward some distance. Moreover, one of the most notable facts about knowledge — and this is plainly connected with the presence of meaning — is that it can be expressed. Knowledge can be shut in between the covers of books and passed on from generation to generation; it can be transmitted from mind to mind by word of mouth; it can be embodied in the intricate formulae of the exact sciences. The symbols which are the outward instruments of this expression must have a close and necessary relation to the thought they convey, and the analysis of these instruments should aid us in bringing the true nature of knowledge into view. It is from this angle that we propose to attack the problem.

II

The most troublesome intruder in the field of the theory of knowledge is metaphysics, for the questions that usually first suggest themselves to the student of the subject are these: How is knowledge related to reality? Is reality independent of or de-

pendent on knowledge? Is it genuinely given in knowledge, and if so how? These questions turn on the distinction of subject and object, of a knowing-being with ideas, perceptions, and sensations, and an external world of existences alien to this being. Is this alienation complete? Or is there a *rapprochement* between reality and the mind in which it makes its appearance? Many philosophers believe that before a theory of knowledge can be stated these problems must be solved.

But if this were so, it would be as if the physicist insisted that a theory of the ultimate nature of mass is essential to a statement of the laws of mechanics, or as if the biologist refused to examine the laws of heredity without an adequate definition of life. Human knowledge is a fact just as mass, motion, and life are facts. The study of its principles, its elements, its structure, does not presuppose an answer to the more ultimate questions, how is this fact possible? — what is the relation of knowledge to reality? These queries carry one beyond the analysis of knowledge itself to a theory of reality, and no one can hope to discover whether or not reality is reached through knowledge until he knows what knowledge is.

Though none of these metaphysical questions is prior to the descriptive study of knowledge, no theory would be complete unless it attempted to answer them. A good theory — of anything — must consider all the intelligible questions its data suggest; and so a complete theory of knowledge will fall into two parts: it will be both a positive analysis and a metaphysics of knowledge, it will both describe knowledge and explain its relation to reality. But the metaphysics of knowledge demands a final concept of reality, and it naturally supervenes on a more restricted inquiry which makes use of a provisional, working concept of the real. Let us call this the *positive theory of knowledge*, epistemology proper, which is an examination of knowledge undertaken in the spirit of the laboratory. We must begin by

viewing knowledge as a natural phenomenon if we are to determine its place in reality. We must proceed inductively and "rise by gradual steps to that which is prior and better known in the order of nature," instead of "beginning at once by establishing certain abstract generalities." [1]

From the point of view of the positive theory of knowledge one can set out without being a realist, an idealist, a monist, a pluralist, a nominalist — without attaching himself to any system of metaphysics. Metaphysics comes later; it is the coping-stone of the theory and it cannot be securely placed without a careful survey of the facts.

The confusion of metaphysics with the positive analysis of knowledge can be traced, in modern thought, to Locke. Locke's *Essay*, though it deals with the structure and laws of knowledge, especially with the manner in which ideas are built up from simple impressions into complex systems, is nevertheless shot through with the conception of an unknowable substance which lies beyond ideas. This conception colors the whole work. For Berkeley and Kant, metaphysics is still intertwined with the descriptive treatment of knowledge, though the *Critique of Pure Reason* in distinguishing the *transcendental aesthetic* and *analytic* from the *dialectic* makes a division that corresponds roughly to these two aspects of the subject. Yet the whole *Critique* is haunted by a metaphysical ghost — the thing-in-itself. The first modern philosopher who severely restricts himself to the analysis of knowledge without metaphysics is Hume, and this is accidental, the result of his scepticism. Since he believed in no theory of reality, he advanced no theory of the relation of knowledge to reality, but he fell on the contrary into the trap of the purely psychological point of view.

The *phänomenologie* which has become prominent in recent German philosophy, especially through the writings of A.

[1] F. Bacon, *Novum Organum*, Bk. I, Aphorism 22.

Meinong and E. Husserl, pursues the analysis of knowledge in the positive spirit; and the same tendency, if not the same matter, is to be found in England in the work of Mr. Bertrand Russell, Mr. A. N. Whitehead, Mr. G. E. Moore, and Mr. C. D. Broad.

Yet metaphysics always lurks in the background, even for those who attempt to avoid this type of speculation. The question of the ultimate validity of knowledge remains to be answered; the notion of truth points inevitably to a final concept of the real. A definition of truth cannot fail to invite metaphysical criticism, and it can be defended in the last resort only by arguments that rest on metaphysical premises. Therefore, unless we are to persist in a partial scepticism such as Hume's, the separation of the positive theory of knowledge from metaphysics cannot be maintained to the end; but it is of immense value as a working distinction in the study of epistemology.

CHAPTER I

MEANING

I

WHAT are the facts that a theory of knowledge needs to organize and explain?

It is customary to say that knowledge is built up from ideas. Locke defined knowledge as "nothing but the perception of the connection of and agreement, or disagreement and repugnancy, of any of our ideas," and he gave as his reason for this definition that "the mind, in all its thoughts and reasonings, hath no other immediate object but its own ideas, which it alone does or can contemplate." [1] If we ask what ideas are, we learn nothing more definite than that they are the mind's objects. Thus Locke's statement that "our knowledge is conversant only with ideas" affirms that knowledge is conversant only with the mind's objects — a truism which no one will deny. The term "idea" is robbed of its force by being extended to all the elements of knowledge.

There is a narrower and more profitable sense of this term, which makes ideas one class of the ingredients of knowledge, but not the only class. This is the sense in which "idea" is used by the psychologist. An idea is an *image*, a psychical event, and it must be distinguished from other psychical events, namely sensations and perceptions. The ground of this distinction is difficult to establish. Intrinsically an image and a sensation or perception are much the same. The difference is not, as Hume supposed, one of vividness or intensity. But the things of sensation and perception have a type of coherence with one another

[1] J. Locke, *Essay Concerning Human Understanding*, Bk. IV, ch. i, secs. 1, 2.

which those of the imagination do not have. Sensory and perceptual objects obey, not merely "psychical" laws, but also the laws of physics; and so in most cases images can be set apart from sensations and perceptions.

The striking thing about ideas in this narrower sense is that they usually refer beyond themselves to other things. They are not as a rule self-contained, but reach out with tentacles of significance toward other ideas or toward the things of sensation and perception. Locke found this to be characteristic of all ideas in his sense of the term, that is, of all the mind's objects. They are all symbolic of that which lies beyond the mind's grasp, says Locke. But since the realities which these ideas signify are not themselves ideas, Locke is at a loss to interpret the meaning of knowledge in terms of these extra-mental realities. He is clear however on one point, that ideas in so far as they are vehicles of knowledge are symbolic; that the area of knowledge is coextensive with the area of significance.

In the more restricted sense of "idea," it is not necessary to attempt the impossible, to seek beyond knowledge for the meaning of ideas. They are to be interpreted as referring to objects sensed or perceived, and often as referring to other ideas or psychical states. But as mere meaningless successions of psychical phenomena following the laws of association, ideas (images) are of no special importance in knowledge; they are not conveyors of knowledge. They make their appearance before the mind and pass away, and this is all that can be said about them. However, as mediators between the mind and objects in perception or objects beyond the immediate circle of the mind—in other words, as significant symbols—images are the first instruments of meaning and hence of knowledge. Instead of saying as Locke does that "knowledge is *of* ideas," one ought rather to say that knowledge comes *through* ideas. It would be of slight value to have knowledge *of* ideas if we did not know *through* ideas.

Images, then, pass by the easiest possible transition to a second level of knowledge, the level of significance. Without perceptible effort, the mind takes them to refer to something other than themselves, and thus endows them with a function which attaches much less readily to other elements of knowledge — the function of meaning.

II

What do images signify? The simplest answer is that they signify things they resemble. Though this is true in many instances, and is indeed the reason why they are such ready vehicles of meaning, images are not restricted in their significance to the objects of which they seem to be faint and partial copies. The vagaries of individual processes of imagery are without limit, and therefore naïve copy theories of the significance of ideas are faulty. The subtlety of the threads that bind ideas to their objects must be recognized. Any image may come to mean any object.

The fact that images resemble the experiences which produced them, and that the image of a part of an experience tends to recall the whole, so that they take on meaning with a minimum of effort on our part, makes images the primal instruments of knowledge. But they are not the only instruments. Man adds to them and improves on them. He adds speech and writing, and finally the complex symbols of mathematics and logic. Ideas take their place as one among many classes of symbols.

It is no doubt possible completely to supplant images as vehicles of thought by words or other conventional signs. Yet, when the major burden of significance is carried by symbols other than images, the latter usually arise in the process. In most minds significant imagery is never wholly absent. The traces of our intellectual childhood, when to think of things is to see them vividly in imagination, remain with us in the most abstract

kinds of thought. Witness the many attempts to visualize Einstein's "spherical universe." And this is so much the case that we tend to believe that the meanings of words or other written or spoken signs are nothing more than images in the mind, either in our own or another mind. Being in the habit of discovering the significance of most signs *via* the route of imagery, we fail to observe that what is meant by words or other signs is not, as a rule, a set of images, but the things for which these images stand. If I say, "The sun is shining brightly outside my window," the reader may believe that when he has called up a picture of sunlight streaming through a window he has before him the meaning of this sentence. But the sentence means neither an image in my mind nor in his. It means *through* these mental events *to* something else. (This does not deny that some words and sentences mean images only; *e.g.*, "my mental picture of China" refers only to ideas in the narrower sense of this term.)

It cannot be maintained that all meanings terminate in images and that when this type of symbol is absent there is no meaning. Images as well as other symbols are not confined in their significance to any class of objects. They may stand for my own or another person's psychical states, for the content of a perception, for other symbols, for objects that have never entered any one's experience; and not only may symbols stand for any kind of object, they may, as will presently be shown, stand for no object and still be significant.

Some objects are, to be sure, more accessible than others. I have a closer knowledge of my own feelings and perceptions than of another person's. The paper on which I write is open to inspection in a way that the other side of the moon is not. But symbols can refer to the inaccessible as well as the accessible; and if there are any objects which symbols cannot mean, these will appear only when we have thoroughly examined the objects they can mean.

The observation that symbols may mean any kind of object frees us from the presupposition that all meanings terminate in the present content of the mind, and at the same time leaves open the question as to the metaphysical status of the objects meant.

III

We have said that images pass readily to a second level of knowledge, that of significance; and that other objects, spoken sounds, written marks, gestures, also take on significance, though somewhat less easily than images.

Below this level of significance (at least of pure significance) is a primary level of acquaintance, and all objects which are significant are also objects with which we are acquainted. Whether the symbol be a sensation, an image, a perception, whether it be believed to belong to the physical or mental world, this symbol will be something *presented*, and will belong to the primary as well as the secondary level of knowledge. This immediate way of knowing, by direct presentation, is the basic form of knowledge, and it has been frequently distinguished from a less direct way — knowing *of* or *about* objects. I am, for example, presented with the groups of letters and words on this page and through them I know about the subject under discussion; when I say that I know the whiteness of this paper, it is clear that I do not use the verb "know" in the same sense as when I say that I know Marcus Aurelius was a Roman Emperor. I am acquainted with the one object but not with the other.

Knowledge about things springs from acquaintance with them or with other things in terms of which they can be described; and though it is difficult to explain what it is to be known in the sense of being immediately presented to a mind, without this sort of knowledge we should probably know nothing. We are here in contact with a fundamental concept of our theory, the concept of the presentation of objects.

Among the objects presented, some take on meaning and become symbols, while others remain merely presented. What is given to the mind becomes an instrument by means of which things not given are represented, and thus we begin to pass from immediate to purely mediate knowledge. To know about things is to refer to them through symbols when they are not presented, while to know things immediately is to have them before the mind, not simply as something referred to, but as something given, in which symbolic references terminate. My knowledge that Marcus Aurelius was a Roman Emperor is exhausted by statements I can make or ideas I can entertain. I can speak significantly of him, I can think about him, but I cannot be presented with him. And this is true of all events in the past and future, of distant places, and probably of other minds. I can, at the moment, reach these only through symbols. On the other hand, my knowledge of the object I call "the whiteness of this paper" is not exhausted by what I can say or think about it. This knowledge is more than a significant idea or statement; it is an immediate presentation.

The most striking fact in knowledge is that it falls into these two divisions, mediate and immediate; that mediate knowledge is wholly symbolic, a reference to objects through objects, while immediate knowledge, even though it may be partially mediated by symbols, is more than reference to objects. It is a unique seizure of objects by the mind. Yet this distinction between mediate and immediate knowledge is not so sharply marked as many contemporary schools of philosophy, especially those which maintain that knowledge is constructed from sense data, would have us believe.

IV

Objects which are presented are not merely sensed. The unit of cognition is not a sensation but something much richer. Sen-

sations are discovered as elements in larger wholes, and these larger wholes are perceptions or *objective presentations*. The datum of the theory of knowledge is much nearer to William James's *stream of thought* than to pure sensation.

"Most books (of psychology)," says James, "start with sensations, as the simplest mental facts, and proceed synthetically, constructing each higher stage from those below it. But this is abandoning the empirical method of investigation. No one ever had a simple sensation by itself. Consciousness, from our natal day, is of a teeming multiplicity of objects and relations, and what we call simple sensations are results of discriminative attention, pushed often to a very high degree." [1]

Empiricists rarely mean by "experience" that which is given wholly through the senses at the moment of experiencing. We *experience* objects, relations, qualities; we *perceive* things of certain sorts rather than of no sort in particular; we are *presented with* situations and facts, rather than with bare sense data. From this full experience certain irreducible elements that come from the senses can be analyzed out; these are elements such as *white, hard, smooth;* but these elements are not the whole of the experience. They are always bound up with other elements.

The cognitive unit, a presentation, is therefore complex. It includes a concept (and often a belief) as well as sensations. If I gaze from my window at the trees bending in the wind, there is much more in my mind than impressions of color, movement, shape, and relative position. I see the trees. My mind leaps beyond sensations to concepts — concepts of solid three-dimensional objects of a certain nature. The fusion of concepts and sensations is the presentation of the object, and neither concepts nor sensations by themselves would give the peculiar kind of cognition I call "presentation." The simplest experience, *e.g.*, that of the color *white* which is now before me, is more than a

[1] W. James, *The Principles of Psychology*, vol. i, ch. ix, p. 224.

pure sensation. I recognize something as *white*, and in doing so bring it under a concept.

Thomas Reid, the leader of the Scottish school of common-sense philosophy, which succeeded Hume, adds that belief also is a constituent of perception. We believe "irresistibly," he says, in the existence of the object perceived. But this is not always the case. An illusion may be present, actually perceived, but not believed; we often doubt our perceptions; seeing is believing only to credulous minds. Though there is a strong tendency to believe our perceptions, this tendency cannot be a constituent of the perception, since perceptions persist when they are doubted or disbelieved. A concept, on the other hand, is a necessary part of a presentation.

The concepts which enter in perception may function explicitly, as when I judge that "this is white," or they may function silently, without conscious judgment, as when I perceive the whiteness. But one knows that he is presented with an object only when he brings it, in some way, under a concept. The fact that immediate knowledge can deceive, that we are as vividly presented with objects in dreams and hallucinations as in other states, bears witness to the presence of concepts in immediate knowledge. It is only because the stuff of perceptions is largely manufactured or elaborated by thought that we can be thus led astray. If there were such things as pure sensations, it could be readily granted that they would not deceive. They could be what they were known to be, and nothing else. But since there are no pure sensations, we must admit that any presentation can be deceptive. The very perception that something is *white* or *hot* or *hard* is subject to error.

"Sensation, then," says James, "so long as we take the analytic point of view, differs from perception only in the extreme simplicity of its object or content. . . . A pure sensation is an abstraction." [1]

[1] W. James, *op. cit.*, ii, 1 ff.

It is evident that the presence of concepts in perception blurs the distinction between mediate and immediate knowledge, and especially is this so when we see what a concept is. We shall find that a concept is a symbol taken with the mental attitude that gives it significance. When we conceive of an object we mean it or refer to it, we entertain the idea or symbol of it; and thus the mechanism of conception is the mechanism of symbolism. Immediate knowledge, no less than mediate knowledge, presupposes therefore the functioning of symbols in a mind.

A mind enters on perception with predispositions or intentions, which determine to a large extent what is perceived; and these predispositions are aroused by symbols — ideas, incipient vocal utterances, word imagery — which are at work in the perception.[1] The sensory elements also operate as signs. When I open my eyes on the room in which I now write, the stimuli present to my senses awaken (through sensations) concepts in my mind; indeed these sensations themselves play the part of

[1] The psychologists of the Würzburg school, in recording their experiments on the thought-process, speak of these predispositions as determined by the *Aufgabe*, that is, the problem to be solved in any given case. Something very like an *Aufgabe* is, I believe, present in any mind engaged in perceiving, and plays an essential part in determining what is perceived. The following passage is quoted from a paper by Dr. C. C. Pratt, entitled "The Present Status of Introspective Technique," in *The Journal of Philosophy*, vol. xxi, no. 9, April 24, 1924: "At the congress for experimental psychology at Giessen in 1904 Külpe made a brief report on some experiments of his own on abstraction in which geometrical symbols, the components of which differed with respect to form, color, and arrangement, were used as stimuli. By means of instructions the observers were determined now in the direction of form, now of color, and now of arrangement. And it turned out that when an observer was under the *Aufgabe* for color, *e.g.*, he could make at best only a very inadequate report on form and arrangement — in some cases he reported that form and arrangement were not present to consciousness at all. The implications of such a state of affairs is far reaching. . . . As far as accurate observation and unequivocal report are concerned, an observer is adequate only to those aspects of a given experience which the determining tendency brings clearly into line with the particular *Aufgabe* of the moment; other aspects of that experience fall at various distances outside of the sphere of immediate observation and hence cannot be made the objects of scientific description." See O. Külpe, "Versuche über Abstraktion," in *Bericht ü. den I. Kongress f. exper. Psychol.*, 1904, pp. 56–69.

concepts, that is, of symbols. They arouse unuttered judgments which are carried by images or fleeting and unspoken words. Through the whole process, I am aware in immediate presentation, not of the sensations alone, but of the objects I thus conceive. I see the desk and the rows of books because these are what the passing sensations mean.

If the presence of concepts in immediate knowledge blurs the distinction between the mediately and the immediately known, still it does not obliterate it. As we travel up the scale from presentation to representation, we come finally to a kind of knowledge in which the object conceived or meant is no longer *given* along with the symbols through which it is conceived. This is the kind of knowledge embodied in a mathematical expression or a scientific theory; or, more simply still, in a judgment as to the past or future. It is purely symbolic. Between presentation and representation there is a twilight zone in which one cannot be sure whether the object intended is given or merely conceived. Here most of the errors of perception occur. But the ends of the scale are distinct. At the one end is a direct acquaintance with objects such as is not possible through concepts (or symbols) alone: this is a cognition in which what is meant is grasped in a union of sensation and conception, often attended by belief. At the other end is pure conception, knowledge which is *significant reference* and nothing more.

V

The statement that concepts (symbols) are always to be found in immediate knowledge must be modified if there is any such cognition as M. Henri Bergson's intuition of the flux of reality. The Bergsonian intuition is a *pure awareness* more immediate than the perception we have described; it does not give presentational knowledge in our sense of the term; it is not an acquaintance with *objects*, not even with simple qualities.

One cannot adequately explain what this intuition gives him. He must search for it in his experience and if he finds it must admit his inability to express it. He will be unable to bring it under any concept, but must remain content to gesticulate in its direction with words and metaphors. I cannot, in Bergson's sense, have an intuition of an object, for this would require the concept of *an object* — the content of the intuition would be conceptualized in this general and vague way as "something-or-other." Thus if we grant the existence of such a non-conceptual form of cognition, it will give us neither objects, qualities, relations, things, or events, but will fall into a category of its own, beyond all rational categories.

Now there does seem to be a background of *pure awareness* not unlike the Bergsonian intuition in all presentations. Beyond the objects which are clearly given, beyond what is singled out and conceptualized as "experience," there is something which cannot be singled out and conceptualized. This is an amorphous datum which transforms itself, under the working of concepts, into the articulated data of perception.

Perceptual knowledge, the knowledge of objects, persons, events, places, relations, qualities, sense data, has a structure. Only after we have examined this structure will the distinction between pure awareness and rational presentation become clear. Pure awareness must be the complete antithesis of conceptual knowledge; it must be knowledge in which all symbols have been transcended, knowledge wholly deconceptualized; and yet it must be continuous with perception, and through perception, with conception. The presentation of objects must arise from this pure awareness and return to it with no perceptible break.

Theories of knowledge which find pure immediacy in sense data do not carry their analysis far enough. The sense datum is something discrete, fixed and referred to by a concept, and be-

hind the sense datum is the vaguely apprehended whole from which it is discriminated. Even so fluid a datum as Mr. A. N. Whitehead's *event* [1] is not known in pure immediacy; the event is picked out from a background, and certainly the perception of relations between events demands more than a purely immediate knowledge.

The attempt to discover the purely immediate leads thus beyond perception and sensation to what can be described only as the unarticulated awareness of a whole. (And even this description is faulty since it makes use of concepts.) But this pure awareness is not a form of knowing which can stand on its own feet. It issues from the whole cognitive act as the final aroma of knowledge, as a sense of oneness with the object known; and the attempt to isolate pure awareness meets with no better success than the attempt to grasp a pure sensation. Though this intuition passes beyond what can be clearly compressed into concepts, it does not contradict but supplements conceptual knowledge. We must leave till later the discussion of this pure awareness and its relations to other forms of knowledge.[2]

VI

From the basic units of presentational knowledge, purely symbolic or conceptual knowledge is constructed. Through symbols, themselves presented and significantly linked to objects (and sometimes to no object), we move from the realm of presentations to that of representations. It is here that the sciences appear, that logic enters, that the common knowledge of everyday speech comes into being. The world of the purely conceptual is superimposed on or abstracted from the world of the immediately known. The mind reaches out from its data. Words, ideas, and other signs carry it beyond things that are given to

[1] A. N. Whitehead, *The Principles of Natural Knowledge* (1919), ch. vi.
[2] See below, ch. VIII, sec. xii.

the imaginary and even to the non-existent, the fantastic, and the impossible. And this brings us to the central problem of the present chapter, the nature of meaning.

A symbol, briefly described, is an object which stands for another object or is still significant if it stands for no object. This is no definition, and serves merely to raise the question of how a symbol is related to the thing it means and how it can be significant when it means no existing object.

In the first place, the symbol stands *in a mind* for an object; it takes on significance through psychical activity, and if there were no minds there would be no symbols. Since the symbolic relation is not something given in the external world, any significance discovered in the events about us is read into these events; and quite apart from the metaphysical question as to whether these events could *be* if there were no minds, they certainly could not be significant unless they were so interpreted. When one says that clouds *mean* rain, that a low temperature *means* ice and snow, he refers to a causal connection between these things, but this relation alone does not make the one a sign of the other. It is because the cause is interpreted as a symbol that the antecedent event takes on significance. Aside from the connections of meaning established in a mind, things in the order of nature simply *are;* they are bound together by laws, but not bound so that any one by its own nature means any other. Berkeley, who interpreted all sense impressions as signs, found it necessary to postulate a Divine Mind communicating with man through this language of natural events.

Written marks and spoken sounds are not different in this respect from other objects: without a mind which uses them to refer to things, they would be nothing more than physical occurrences; and the same is true of images or ideas—unless we took them to be significant, they would pass through the mind as psychical events, conforming to the laws of association but bearing no meanings with them.

Meaning then is something superadded to things by a mind. What is the nature of this activity which adds significance?

The commonest answer to this question, familiar especially in English philosophy from Hobbes onward, is that the meaning activity is the association of ideas; and this reappears in James, who says that the meaning of an idea is to be found in its "psychical fringe." Locke sees the weakness of this view when he speaks of association as a distemper of the mind which leads thought away from its objects rather than toward them. In determining the meaning of a word or image, we must rule out irrelevant associations; much of the "psychical fringe" must be overlooked; and this being the case, meaning must be something other than association. Though associations sometimes conduct one to the things he means, they more often conduct him away from them, and he can never be sure that the paths of association are not by-ways rather than high-roads of thought. The associations that play a part in carrying meanings are controlled, that is to say, their direction is prescribed by what the psychologists know as a "determining tendency" or an *Aufgabe*. The word "New York" may arouse by association the image of the Statue of Liberty, which may lead to a picture of the guillotine and the red flag, but none of these associated ideas is the meaning of "New York"; they must be dismissed as irrelevant associations. Images of canyon-like streets and suspended bridges bring the meaning nearer, since they fit the attitude or set of mind the word induces.

There is a still more damaging objection to the view that meaning is the association of ideas. Association is a link between images and carries the mind no further than ideas in the narrower sense of the term; and yet what is meant by a symbol need not be an idea. "New York" means *the city on the banks of the Hudson*, rather than the representation of this city in images, and these images themselves mean something other than images.

They stand for a perceptual object, and the word means what the images mean. The activity of meaning is, more frequently than not, directed beyond ideas, and so it cannot be described in terms of association, which is restricted to ideas.

VII

The simplest solution for the purposes of the theory of knowledge is to accept as unique a *meaning activity*. This does not imply that from the point of view of psychology this activity is unanalyzable; it may well be that it can be reduced to more elementary activities, but it is not necessary for our present purposes to do so. We must confine ourselves to a general description, which is intended to direct the reader to the point in experience where the meaning activity is to be found. Having discovered meaning in its primitive form, we can show how more complex meanings are constructed, and this will lead to the logical and strictly epistemological, rather than psychological, aspects of the subject.

To reach out with the mind toward objects, as one does when he means them, is to be in a state of preparation for these objects. The meaning activity is one of vague anticipation: the mind is poised expectantly, awaiting something other than the thing, the symbol, which is immediately before it; and this anticipation is *vague* because it is not accompanied by a belief that the object meant will appear or that it exists. When I mean an object I do, in some sense, prepare my mind for a presentation of this object. Though I cannot be said to turn my attention toward the thing I mean, since one cannot attend to something not presented to him, there is no doubt that I do more than attend to a symbol or an image. Indeed, I turn my attention away from the symbol or the image, and this constitutes the first step in preparation for the thing meant.

Toward every object certain activities are appropriate. I can

eat bread; I can throw a stone; I can sit on a chair. These activities are appropriate to the object for many reasons, the most important of which are biological reasons: the appropriate activities are those which enable me to adapt myself to the thing; and for any single thing there are many such activities. When a symbol is before the mind, it sets in motion the activities appropriate to an object and this object is then what the symbol means; but it sets these activities in motion only partially, it touches nothing more than their psychical roots, so that if the effect of the symbol appears in consciousness, it is as a barely noticeable feeling of tendency.

Physiologically, a symbol probably stirs the central part of a chain of nervous connections which constitute a tendency to act in a certain way. The process of understanding a symbol terminates in the brain, and for this reason the activities it arouses are implicit, so implicit as to diminish almost to the zero-point both for outward and inward observation. And yet they are adequate to carry the reference, the outward reach of the mind, which is significance.

Ideas, as well as written and spoken words, set up these central tendencies to action. Every idea, in James's opinion, has its motor side; and not only do ideas arouse tendencies to action, they spring from these tendencies, so that the set of the mind in a certain direction determines what images appear and causes irrelevant images to be disregarded. It is this set rather than the image itself which fixes the meaning, and this is why ideas do not mean by resemblance alone.

But what if the activities brought into play by a symbol become overt? What if they push out beyond the central nervous system and terminate in completed acts?

In this case the mind has gone further than merely understanding the symbol. Understanding has passed into belief, that is, one has begun to behave as if the thing meant were present,

as if the activity which is fitted to it could be successfully performed here and now. Understanding and belief being of the same genus, the line which separates them cannot be sharply drawn. Belief is willingness to act on what is understood, while understanding is preparation for activities appropriate to an object, though these activities are checked far short of performance.

If you tell me that it is a good day for a walk in the hills, I understand by putting myself in an attitude of mind suited to a walk in the hills, but this attitude does not amount to belief. Though I need not actually set out on the expedition if I believe the statement, I must nevertheless be more fully prepared to do so than if I merely understand it. I must feel that the walk could be successfully undertaken. Belief is a more complete preparation for activity than understanding: it is understanding with an added psychical pressure in the direction of performance. Therefore, if the effect of a symbol on a mind is to produce overt acts, we can conclude that the symbol is not only understood but also believed.

VIII

This is not the only possible conclusion, and here the inadequacy of the behavioristic treatment of meaning comes into view.

The extreme behaviorist assumes that there is only one way in which psychical processes can be studied, namely through outward action. Now an object that is immediately presented may produce outward activity without either understanding or belief. Certainly if one is to believe, he must begin by understanding, for he cannot believe what he does not understand even in a vague sense; and so if overt activity does not testify to understanding, it cannot prove the presence of belief. The crucial question in the behavioristic account of thought is, then, does overt activity indicate understanding?

Let us suppose that our dog comes for his food when the dinner-bell is rung. The bell has aroused an activity appropriate to an object, and we can assume that the bell *means* dinner to the dog; and yet we cannot be sure of this. The dog's act is not different, so far as we can know, from many other habitual acts. He will go to a certain corner each night to sleep, he will bark at certain persons and not at others, he will howl when the moon is full. Are we to say that all of these acts are acts of understanding?

The weakness of the behavioristic theory of meaning is that it affords no criterion by which acts of understanding can be distinguished from other habitual acts, yet no one would be willing to say that all such acts are cases of understanding. The theory leads us to the conclusion that any stimulus to which we react in an habitual way is a symbol.

It is true that most signs acquire significance through habit. The connection between the sign and its meaning is based on a conditioned reflex. The object A, food, for example, stirs the activity B, hunger and eating; and if X, the sound of the dinner-bell, accompanies A for a sufficient number of times, X alone will become an adequate stimulus for these activities. This is the pattern of all habits. But there is a distinction between habits which carry meaning and those which do not. Though we should have no speech and writing were it not for our ability to form habits, the formation of a habit does not argue a connection of meaning; there are many habits in our behavior that are not vehicles of significance.

Habits of meaning belong to a special class. They are abbreviated habits, habits purely of the mind, not extending beyond the central nervous system. When we understand a word or an idea, we perform no observable acts, unless it be silently to repeat the word. The actions appropriate to the object are confined to the brain, or nearly so; and it is just this mental or inner character

of habits of meaning which differentiates them from meaningless habits. A sign causes us to *think* rather than to go off into a series of immediate and unreflective acts, as do other habit-stimuli. Deliberation, abbreviated action, understanding, rather than overt activity, are the effects produced by symbols.

The behaviorists explain that meaning is based on language habits and that meaningless vocal acts or habits develop into language habits, which carry significance when they become associated with "arm, hand, and leg activities and substitutable for them." [1] This is an attempt to formulate a criterion of significant as opposed to meaningless habits. The *substitution* of a sign for an object indicates that the sign is understood, that is, that it has become a sign. But substituted how? Substituted where? And here the whole problem of meaning is concealed. The only intelligible answer is, substituted in an internal experience not open to external observation.

It is impossible without introspection to say when this substitution has taken place. Mr. J. B. Watson's example of such a substitution is inconclusive: the fact that sounds are uttered in conjunction with arm, hand, and leg activities,— in conjunction with an infant's reaching for an object held out toward him,— and that the sounds alone are uttered at a later time, either when the object appears or when it is absent, does not show that the sound has been substituted for these activities, that the child has passed from unreflective action to deliberation, and that the sound has come to *mean* the object. The substitution of a symbol for an object, which is its "standing for the object," is essentially a fact for introspective observation and only secondarily a fact of behavior.

Meaning is an activity that can be described only in introspective terms, and even in introspection it is difficult to grasp.

[1] J. B. Watson, *Psychology from the Standpoint of a Behaviorist* (1919), pp. 319 ff.

Thought is distinguished from action simply on the ground that it is activity which does not work itself out in behavior. If thought is behavior, it is behavior inhibited and yet fully aware of its directions and intentions. The habits on which it is built are private. Though it may be true that the dog genuinely thinks dinner at the sound of the bell, we are not justified in inferring that he does from his behavior alone; and if our own reactions to dinner-bells and to words or images were always overt and fully carried out, if we could not pause to deliberate and so check, short of performance, the tendencies to action aroused by symbols, we could not be said to understand language or ideas, but only to react to them as we might to a bright light or a loud noise. The moment of suspension of mind between the immediate stimulus and the activities that follow would be shortened to exclude understanding, and none of the stimuli of habitual actions would become symbols. They would remain merely objects which set in motion conditioned reflexes, and their capacity to mean other objects would be as yet unrealized.

No doubt there is an insensible gradation between habits that do not carry meaning and habits that do, and by this gradation an organism passes from action to thought. As the capacity to retain the effect of a stimulus in the mind without overt activity increases, the ability to understand is gained, and the objects to which the organism has before reacted habitually without understanding become symbols of the things toward which these activities are directed. Unless this capacity to suspend reactions and to anticipate an object rather than to behave as if it were present were developed, we should not be able to think but only to act.

IX

The psychological aspects of meaning can be summarized as follows. A significant reference to an object with an object is accomplished through a suspension of activity, known in intro-

spection as an *intention*. A symbol touches a train of psychical connections which, if followed out, lead to activities appropriate to an object. In understanding alone, this train of connections is remotely stirred and is not pushed to its terminal point, but when the suspension of mind breaks over into the actions suited to the object, understanding passes into belief. Belief must be preceded by understanding, and overt activity is not necessarily a sign of belief or of understanding. In this suspension of activity the mind reaches out beyond its immediate field, and the stimulus of this outward reach, whether it be an image, a word, a mark, or a gesture, is a symbol.

A general conception of what it is to think can now be framed. Without symbols there is no thought. Thinking is activity, or rather the suspension of activity, through symbols. It is the substitution of symbols, which are in the field of presentation, for the things intended by the symbols, which may be beyond this field; and sometimes thought is carried by symbols that stand for no objects. Thought is not to be classified with sensations, images, feelings, conations, as an element of consciousness. It is built up from these elements, and it may be from others; it is the use of these elements for the purpose of recording and conveying significant references.

If psychologists have discovered an imageless thought, they have not discovered a thought which does not rest on the use of symbols. Symbols are the universal instruments of knowledge, and the ancient logicians who defined man as a "rational animal" might well have given another definition, "employer of signs," for man's rationality consists in his ability to use signs. Without the semi-artificial media of language, writing, and the numerical system, human knowledge as we know it could not have come into existence; and beneath these systems of semi-artificial signs lies a still more primitive system, that of images.

To indulge in pure thought is to put in the place of the things

found in experience a set of substitutes, secondary realities such as marks on a page, vocal noises, psychological images, and to move in this world in a way corresponding to that in which one might have moved among the primary realities. And thought is more than this: it is the extension of the references of symbols beyond the experienced, it is the significant use of signs when there are neither presented nor existent objects for which they stand. These more complex types of meaning remain to be considered.

X

It is not difficult to see how things such as colors, shapes, sounds, simple states of mind — anger, fear, pleasure, pain — can be referred to; but one speaks significantly also of "the universe," "the world," "the solar system," and yet none of these objects has entered experience in its wholeness, nor is there any single activity or set of activities appropriate to it. This is true also of more restricted objects such as "New York City" or "the British Empire." We cannot mean these things merely by taking a single symbol to stand for them. The mind must reach out toward them in some immensely more complex fashion than it does toward simpler objects.

All of our most complex ideas, says Locke, even to the idea of God, are constructed from simpler ideas which are the foundations of knowledge; and Locke's distinction between simple and complex ideas can be extended to all symbols. Objects such as the universe, the other side of the moon, the continent of Europe, are referred to by means of *complexes* of references to simpler objects.

Most of the things with which we are familiar are easily seen to be composed of other things, they can be separated into elements; and if an object as a whole cannot enter experience to become the terminus of a meaning, some or all of its components usually can. Though we cannot directly form habits of speech or

thought appropriate to the universe or to New York City, we can come in direct contact with parts or aspects of these objects. These objects can be analyzed and thus can be represented in thought; the whole can be grasped through its elements.

The symbolic systems of language, mathematics, and the imagination are peculiarly adapted to this *analytical representation* of objects. Men think not by means of isolated words, signs or ideas, but by joining these into groups, into propositions, phrases, and sentences, which have a meaning as a whole. This is *syntactical meaning*. Syntax is literally "taking together," and symbols taken together are significant, as symbols by themselves cannot be: any phrase, sentence, or complex idea — any group of symbols — has a meaning other than the meanings of its elements but determined by these, and these alone. This gives us the first principle of symbolism, the principle of syntactical significance: *the significance of any group of symbols is a function of the significance of its members.* To the elements of a symbolic group correspond elements of the object which might be meant by the group as a whole, and thus, through a symbol that can be analyzed into parts, an object that can be likewise analyzed is represented. The sentence, "the sun is shining outside my window," means a fact composed of the objects meant by "sun," "shining," "outside," "my," and "window." The fact is these elements, built up according to a definite plan, taken in a certain order, and the meaning of the sentence is a synthesis of separate meanings in a single meaning.

XI

Before we can employ groups of symbols to represent objects analytically, we must be equipped with simpler instruments of thought. These are *simple symbols*, whose significance is of the direct sort described above.

Simple symbols have no syntactical meaning. Their signifi-

cance is not a function of that of their parts, if they have symbolic parts, but rather a function of the presentation of objects and of a mind which establishes connections of meaning between presented objects. The significance of "Shakespeare" is not determined by the meanings of "shake" and "speare": it arises from the direct use of this name for a person. (A simple symbol may have meaning, however, through definition only, that is, it may be defined as equivalent to a symbolic group, *e.g.*, "nectar" is "the food of the gods"; and in this case the meaning will not be a function of the presentation of an object for which the symbol is taken to stand.) The conditions under which simple symbols acquire meaning are such that they cannot mean objects which have not appeared in perception, unless they are expressly defined through symbolic groups which mean such objects. Knowledge is built on experience, on presentation, because it is only in experience that these first instruments of knowledge are of use. In order that (undefined) simple symbols may take on meaning for us, we must have performed acts directed toward the things they mean. We must have experienced the symbol with the object, and a conditioned reflex must have been established, so that when the symbol appears the appropriate activities and the intention directed toward the object will be set up.

It was said above that without concepts, that is, significant symbols, there are no presentations of objects, and it should be added that without presentations of objects there are no concepts. Simple symbols, that is, single words or ideas and the psychical attitudes which accompany them, play a necessary part in determining the cognitions of their objects, and at the same time the cognitions of these objects play a necessary part in determining the meanings of the simple symbols: they are functions of one another, and it is impossible to say that the presentation of objects precedes in time the use of symbols. Perception is born through the significant use of symbols, and the significant use of symbols, through perception.

The significance of a simple symbol, unless it is defined through a symbolic group, rests on three necessary factors: first, the symbolic object proper, the mark, the image, the sound; then the attitude of preparation, the intention or psychical set; and finally the presentation of the object meant. To say that a concept enters in every perception of an object means simply that these three factors are always present in perception. I am never presented with a thing unless I am also presented with a sign appropriate to the thing. Significance of this simplest sort is a part of every experience. Symbols and the mental attitudes which accompany them pick out things, relations, qualities, sense data, from the background of pure awareness and make them objects of knowledge. Signs are a genuine part of presentations, and we come originally to know their meanings apart from presentations by separating them out, rather than by artificially creating and applying them. The artificially created sign is a late product in the evolution of thought. Having been sifted out from the presentation of which it was in the beginning a part, the sign can *mean* when the object meant is not presented, but it could not mean this object unless the latter *had been* presented.

It is less evident that words are integral parts of presentations than that images are. Words probably appear later than images as carriers of cognitive attitudes. But theories of the development of language assume that words were originally bound up with perceptions. Those who believe that language arose, in part at least, from ejaculations say that a noun such as "ache" comes from the ejaculation "ach!" the cry of pain; the pronoun "me" from the ejaculation "ahem!" by which the speaker involuntarily calls attention to his own presence.[1] Cries and vocal expressions, once parts of an experience, were thus cut off from the whole and became significant in their own right. Certainly

[1] G. Willis, *The Philosophy of Speech* (1920), p. 9.

in our adult perceptions we are often conscious of the passage of words through the mind as a means by which we adjust ourselves to the object perceived.

There can be little doubt that all perceptions include a psychical set toward the object. The mind is not purely passive in perception, as Locke supposed — not a *tabula rasa* on which experience writes. Perception is an activity, and mental activity is always directed toward something; it always intends something. This active attitude of intending, which is the second essential of simple meaning, is more prominent in perception than the first, that is, the symbol proper, to which this attitude is attached. But the first is nevertheless present. The symbol proper arises somewhere in the course of the perception, and the three elements, symbol, attitude, and object, are united in a whole which is the presentation of the object.

Thus, as far back as we can trace our articulated experience, simple concepts are present in it, and these have meaning only through experience because they are of the very stuff of experience.

These elementary symbols (concepts) stand for data. When a single word, gesture, or sound has once been taken as significant without definition, a datum has been accepted; and though subsequent use of the symbol may lead one to observe that the object it refers to is complex and can be analyzed into new data signified by other symbols, still no analysis can be pushed so far that all data disappear. But it must not be forgotten that many simple symbols have meaning only through definition. "Santa Claus" means the same as "the good saint who fills up our stockings at Christmas"; "the universe" is a name for "the totality of objects, known and unknown"; "matter" a name for "objects that obey physical laws." Such defined symbols are abbreviated forms of more complex meanings. They are derived concepts whose objects are not data.

XII

Having once at our command a stock of the simpler instruments of meaning given in our commerce with the immediate environment, we can construct new meanings determined by the old. Through these the imagination takes wings and the world of speculation is opened.

A symbolic group, if it stands for anything, will stand for a complex object, which may be the kind of object signified by a descriptive phrase such as "the center of gravity of the solar system," or the kind signified by a completed sentence either asserted or unasserted, *e.g.*, "knowledge is power." The latter type of symbolic group is ordinarily termed a *proposition*. The peculiarity of a descriptive phrase is that through a complex of predicates or relations it signifies a term, while the meaning of a proposition, on the other hand, is not focussed on a single term.[1]

The constituents of the complex object meant by a symbolic group are signified by simple symbols, and it is clear that these constituents must have been given in experience or must be definable through symbolic groups whose simple symbols refer to elements given in experience. "New York" means "the city at the mouth of the Hudson," and if the objects meant by "city," "at," "mouth," "of," and "Hudson" cannot be known directly, they must be defined in terms of objects that can be known directly; that is to say, every symbol (simple or complex) must signify a datum or be reducible to symbols which signify data. And yet a descriptive phrase or proposition whose simple symbols refer to data or are defined in terms of references to data, may *as a whole* stand for no object; it may state no fact, describe nothing, and still be significant. Thus the phrase, "the hereditary monarchs of the United States," has meaning though it describes no one; the proposition, "the gods live on nectar and ambrosia," states no fact, yet it is significant.

[1] See below, ch. IV, for a discussion of this distinction.

Both reason and imagination rest on this possibility of using symbols significantly when there is nothing to which they refer — a possibility which arises through joining simple symbols into groups. The meaning of the group is a function of nothing but its subordinate meanings and — what is more important still — of its logical form. No object meant need be presented; the being, either in or out of experience, of an object to which the group as a whole refers is irrelevant to the meaning, for group-meanings are determined by meanings alone.

Many writers dismiss the possibility of thinking or speaking significantly when there is no object thought or spoken of, by a simple argument. To think when there is nothing thought of is not to think; to mean when there is no object meant is not to mean.[1] It is denied that symbols can be used significantly when they do not refer to existing objects. This is indeed true of undefined simple symbols but it is not true of symbolic groups. The argument rests on the assumption that all meaning is simple reference to an object with an object, and the more subtle type of reference through groups is neglected.

Syntactical or group-meaning is, like all meaning, an outward reach of the mind, but it takes an indirect course and may never terminate in an object. A phrase sets up all the anticipations or intentions its single words arouse; when I understand the significance of "a sunny day in spring," I am stirred psychologically as I would be by "sunny," "day," and "spring"; I take an attitude appropriate to the things meant by each of these words;

[1] Plato puts this argument in *The Sophist*, Jowett's translation, marginal page 237:

Stranger. You mean by assenting to imply that he who says something must say some one thing?

Theaetetus. Yes.

Stranger. Then he who says "not something" must absolutely say nothing.

Theaetetus. Most assuredly.

Stranger. And he who says "nothing," is not to be described as speaking; and therefore he who says "not-being" does not speak at all.

but my attitude is at the same time a *unity*. I anticipate these elements as forming a whole, and if no such whole exists the attitude of mind will still carry the meaning. Through the many references of its members (and its logical form) the symbolic group, as a unity, has a reference, that is, a direction toward something; and even though there is (or has been) in experience no object for which the expression as a whole stands, there will nevertheless be a *locus* in experience toward which this unitary reference is directed. This is the locus of the objects meant by the elements of the group. There is no "present king of France"; and yet this phrase tells me where to seek for the object it might mean. A symbolic group means through a complex intention constructed from simpler intentions, and so performs the function of *referring*, which is the essence of meaning, when as a whole it refers to nothing.

Syntactical meaning — a meaning of the whole determined solely by that of the parts and their plan of unity — can be found in any phrase, sentence, mathematical expression; in short, in any group of symbols. Its requisites are: (1) a number of simple symbols referring directly to objects, or defined in terms of such references, and (2) a unity of these symbols and the intentions which attach to them in a single intention. It is not necessary that this complex intention should arise from or terminate in any single object.

The manner in which the simple symbols are joined into a unity gives the expression a structure, and it is in the analysis of this structure that the central problems of the theory of knowledge appear. It may seem to the reader that the structure of symbolic groups is merely a matter of grammar. What possible light can the study of grammatical forms throw on knowledge? The fact is that grammatical forms are reflections of logical forms; and if the study of the structure of symbolic expressions is grammar, it is what has been called "philosophical grammar"

— an analysis of the structure of thought. In order to understand what is meant by the form of a symbolic complex, it is necessary to probe deep into the nature of knowledge and its objects. But before we turn to this topic, let us put together the facts of knowledge which are now before us.

XIII

The objects of knowledge are not ideas, unless "idea" be taken in the broad sense of any known object. Ideas in the sense of "images" can be distinguished from sensory and perceptual objects, and the objects of knowledge may be either images or sensations and perceptions. Ideas in the narrower sense occupy a special place in thought: they are the most primitive type of symbols, the primary carriers of meaning. Ideas need not signify merely the sensory or perceptual objects they resemble: any idea may mean any thing. Moreover, semi-artificial signs such as those of speech and writing supplement images, and in some cases completely supplant them. It is important to observe that no symbols are restricted to meaning a particular class of objects, least of all to meaning ideas in the narrower sense.

The study of the principles of symbolism, which is also the study of the mechanism of conception (for a concept is nothing more nor less than a significant symbol), provides a single point of view from which to approach knowledge.

Knowledge as a whole falls into two great divisions: mediate and immediate, knowledge *about* and *acquaintance with* objects. The former rests on symbols alone; the latter is a union of symbolic knowledge and direct apprehension. The presentation of objects, that is, experience, is always attended by the functioning of symbols (of concepts) in the mind, and *intending* or meaning an object is a part of every experience. The mind is not a *tabula rasa* but an active agent in perception, and so the line between mediate and immediate knowledge cannot be sharply drawn.

But immediate knowledge is not sensation, for sensations do not appear by themselves but only as elements in larger cognitive units, in which they usually play the part of signs. All so-called "immediate" knowledge is partly mediated by concepts, with the exception of *pure awareness*, which is the complete antithesis of conceptual knowledge and the only form of pure immediacy. Pure awareness is not the cognition of objects, relations, qualities, or sense data: it is the apprehension of an unarticulated datum. Pure awareness cannot be dissociated from the act of knowledge as a whole.

Signs have meaning only through the activity of minds. Natural objects, even images, are not significant unless they are taken to be so by a mind. The meaning activity is a preparation for the object meant, but a preparation which goes no further than the central nervous system, and which appears in consciousness as an *intention*. Belief, which is of the same genus as understanding, is a completer preparation for an object: it is willingness to act as if the object were in existence. Understanding arises only when the mind is able to suspend the activities appropriate to a thing, and to deliberate rather than act; and though meaning rests on habit, it is not the association of ideas, nor is it the type of habitual response which leads at once to outward action. Thus it is impossible to describe meaning wholly in terms of behavior.

There are two types of meaning, one of which is built on the other: (1) simple and (2) syntactical meaning. Simple symbols are objects taken by a mind to stand for other objects: their significance is not a function of their parts and, unless their meaning is defined, they have meaning only through direct reference to things which are (or have been) experienced. The significance of these undefined simple symbols rests on direct experience because they are an integral part of the presentation of the object to which they refer.

Complex or *syntactical meanings* (so called because they are created by taking together simple meanings) are functions of other meanings; and this sort of significance, which attaches to any phrase, sentence, or mathematical expression, rests on the general principle of symbolism, that the meaning of a symbolic group as a whole is determined by the meanings of its elements. A symbolic group represents an object analytically, that is, as a unity of elements with a form; but a symbolic expression may be significant when it stands for no object, for its meaning is a function of meanings only and not, as in the case of an undefined simple symbol, of the presentation of an object.

Now that we have a general notion of what meaning is we can enter on a wider problem — that of showing how the structure of symbolic expressions points beyond itself to the structure of the world of fact. The garments in which knowledge is clothed, loose and ill-cut as they often appear, reveal the outlines of the form beneath them; syntax and grammar, which seem to be conventions of human intercourse, spring from the adaptation of the mind to an order which pervades its environment.

CHAPTER II

LOGICAL FORM

I

MERELY to place a number of symbols together at random does not make them a symbolic group. The phrases of language are not chance combinations of words; they have a structure, and certain word-structures are significant while others are not. The same words in one combination mean one thing, and in another combination, another thing. The problem of the structure of significant phrases cannot be solved by showing that any group of words which conforms to the rules of syntax in language is significant, for this leaves the rules of syntax to be explained.

Every symbolic system, whether it be speech, mathematics, logic, or the system of mental images, has its own rules of syntax. Within the system some combinations of symbols are significant and others are nonsensical. Some of these principles are arbitrary: they have grown up through usage, as have the meanings of the individual signs, and usage can alter them; but they are not all arbitrary. The structure of every symbolic system is the same in general outline; it is modeled on the structure of fact, for every symbolic system, if it represents anything, will represent facts.

The meaning of a symbolic group is, for this reason, not purely a matter of choice, as is the meaning of a simple symbol. There are certain objects that the group can signify, and other objects that it cannot signify. The structure of the group furnishes a criterion which enables us to determine its possible meanings. At first sight it might seem that groups of symbols

could represent anything, as can simple symbols. Simple symbols are attached to their objects only accidentally, through habit or definition. We can change our conventions for their use as we please. Why is this not also true of symbolic groups? Are not all symbols arbitrary and accidental in their meaning?

If this were so, the problem of how symbols represent objects would be speedily solved: we could say merely that we are in the habit of using such and such signs for such and such things, (though we should have difficulty in *saying* this), and the study of the principles of symbolism would be reduced to the compiling of dictionaries. But this is not all.

When a group of symbols takes on a meaning through the meanings of its constituents, it becomes in a far-reaching sense a representative of an object, if it stands for an object. The relation between a group of symbols and the fact it signifies can be likened to the correspondence of a map to a region of which it is a partial picture. Each element of the map means a feature of the landscape, which is related to other features as the map-elements are related to one another. So, each symbol of a group means a part of the whole fact signified, a part which is related to other parts of the fact as the symbols are related in the group. A phrase, a sentence, a mathematical expression, or a complex idea is as much a picture of a fact as a map is of a country. The difference is that the portraiture is infinitely more subtle. The map reproduces the spatial relations of the country by spatial relations on a smaller scale. The phrase or sentence reproduces the logical relations of the elements in the fact by similar logical relations among the symbols.

A phrase or sentence cannot, then, be arbitrary in meaning. It can mean only a fact logically constructed as it is constructed. Its form must be the form of the fact for which it stands.

It is not difficult to discover in what respects a photograph or a map resembles its original. The correspondence of the copy

with the original is written on the face of both. But the evasive features of logical form are not readily made out. What can there be in common between the sentence, "the sun is shining outside my window," and the fact which the sentence means?

II

Symbols, whether they be words, gestures, or ideas, are objects. In their rôle as instruments of meaning they are still things. A spoken word is a combination of noises distinct from other noises. A spoken sentence is a collection of combinations of noises with pauses and inflections to indicate its grouping into phrases. A written sentence is a collection of marks on a page, distinct from other marks and other collections.

The nature of symbols as objects is not in the foreground of perception when they are used to convey meanings. We are only dimly conscious of the print when we read; we pierce through it, as through a luminous medium, to the meaning. The quality of a voice, the sound of a spoken word, fade into the background of the mind when we attend to a speaker; we hear only what the speaker intends to say. And yet the nature of symbols as objects is not completely forgotten in interpreting them, for it is this that causes them to be recognized as having a structure. There are characters and relations which belong to all objects alike and are the most abstract features of their being, and it is these characters and relations which the symbols — being also objects — reproduce.

Logical form is the least common factor of all objects. Any entity which is presented or thought exhibits logical form, and a thing without logical form (if it could be a "thing") could not be brought within the circle of reason. Where logical form ends, the presentation of objects, analysis, representation, the rational processes, also end; and if any knowledge could remain, it would be knowledge by intuition — the pure awareness which gives us no objects.

The characters and relations that determine logical form appear when we ask what it means to be an object or a fact. "Object" is the most general of all terms. It applies to everything, whether simple or complex; and a definition of "object" is impossible, since it could not be framed without making use of this term. A person, a landscape, a mental state, a quality, a relation, an operation — all are objects. The very statement of the fact that everything is an object is a tautology.

"Object" is more inclusive than Aristotle's "substance" (that is, his "first substance"). By "substance" Aristotle meant an individual, which could be a subject of predication or could stand in relations to other substances. Objects include not only subjects of predication and terms of relations, but also the predicates and relations which can be attached to subjects and terms. Several kinds of objects go together to make up the thing meant by "Caesar was the noblest Roman of them all." *Caesar* is an individual subject; *Roman* is a predicate; *noblest of them all* is a relation — a different kind of object from either of the first two. Each of these, as well as the whole fact, is an entity with a distinct being; each is an object.

To assert that a thing is an object is to make the minimum statement about it; it is to affirm merely that *x is x*. But trivial as this assertion appears, it is implicit in all knowledge of objects; for it informs us that something is singled out, fastened on by the attention, and referred to through symbols.

A fact, which could be represented analytically in a symbolic group, is not a bare "this," an *x* with no character in particular and no relations to other objects. A fact is a complex of different kinds of objects: it is a thing with characters, or a number of terms in relation; it is a concretion of objects. The whole for which "Brutus killed Caesar" stands is a fact. "Rome was not built in a day," stands for another fact. "Fairies have wings," if it stood for anything, would stand for a fact. Taken as wholes,

as clusters of objects, facts are themselves objects of an articulated sort. Unanalyzed objects are the atoms of thought and facts the molecules; but molecules can play a part not unlike that of atoms, and any given fact can become the foundation of more complex facts.

III

The first element of structure in a fact is that it is a *group* — a group whose character as a whole is determined by the nature and relations of its parts. This concept is fundamental in the description of logical form.

A group, as the term is used here, is any object constituted of other objects. The fact meant by (*2* + *2*) is a group, because the whole is a function of the objects meant by *2* and (+). The object signified by "the blue sky" is a group determined by the objects meant by "blue" and "sky." ("The" does not stand for a constituent of the fact.) [1] Similarly, "*a* precedes *b*" means a group — a fact constituted of the objects *a* and *b* and the relation *precedes*. Some groups are subjects qualified by predicates, others are terms in relation, and still others, terms joined by mathematical or logical operations. In short, any object determined by other objects is a group.

The concept of grouping is the same as that of *function*. To say that a whole is a function of its parts is to call attention to the fact that it is a group. Every whole is a group and every group is a whole; and not only is every fact a group, but every collection of symbols with syntactical meaning is also a group, whose character as a whole is determined by the nature and relation of the parts, and whose significance as a whole is a function of the significance of the parts.

It must not be supposed that a disunified plurality of objects or symbols is a group in the true sense. There must be a determination of the whole by the parts. The *relations* between

[1] See below, ch. IV, sec. v.

the parts, if there are relations other than the general unity of the group, are themselves parts or members of the group. Thus *a between b and c* is determined not only by *a*, *b*, and *c*, but also by the relation *between*. This determination of the whole by the parts is essential: without this, the plurality of objects is not a group. A factual group must be a single fact and a symbolic group a single symbol.

The concept of grouping then presupposes, (1) a plurality of objects (members), which may be terms, or characters and relations, or both; and (2) the union of these so that the members uniquely determine a whole; and in the case of symbolic groups, (3) a union of the meanings of the parts so that they uniquely determine the meaning of the whole.

What there is in common between a phrase, a mathematical expression, or a complex idea, and the fact it might represent, now begins to emerge. Both the symbol and the fact are groups. Symbols reproduce groups by groups, as the map reproduces spatial relations by spatial relations. But this gives only the frame-work of the picture. There are an infinite number of different symbolic groups and different factual groups, each of which can be distinguished by its logical form alone. The possibilities of reproducing different structures of fact by formally distinct symbolic structures are without limit.

IV

Every entity, simple or complex, is identical with itself and distinct from other entities. Numerical identity and diversity, together with grouping, determine logical form because they are the most abstract, that is to say the "formal," characters of objects. Differences in logical structure are numerical in nature; they are differences in the number of constituents in groups, and in the number of groups which are members of groups within a whole. Form is in a literal sense number, as the Pythagoreans be-

lieved. *If, in the apprehension of a fact, everything is disregarded excepting the numerical identity and diversity of its elements, and their grouping into wholes, what remains will be the logical form.*

Since a simple symbol is not a group, it is not distinguished from other simple symbols by its form, though it is distinguished formally from any complex symbol. These most elementary of signs differ only materially in meaning from one another, that is, through the concrete characters of the intentions which attach to them or of the objects they mean. At the lowest stage of analysis formal distinctions of significance do not appear, *e.g.*, proper names are all of the same form.

Any analytical representation of an object carries its analysis to the limit of the things meant by the simple symbols, and if any further analysis is possible, this analysis is not contained in the representation. When it is said that the fact meant by "the sunshine on the ocean" is composed of the simple elements, *sunshine*, *on*, and *ocean*, these elements are not affirmed to be absolutely simple, to be irreducible. They are the elementary components of the fact as it is here expressed; the representation goes no further in analysis.

It is evident that the constituents of most facts, as they are expressed in symbols, can be further analyzed. If there is a final limit of analysis, if some data are absolutely simple, most representations stop short of this limit. A simple symbol represents an object with the smallest possible amount of analysis — with a zero analysis. These symbols set the scale of reproduction. Just as the map may employ one inch for ten miles, so a symbolic expression may use a simple symbol for something which is in reality complex; just as another map may depict the same region on a larger scale, so another symbolic group may represent as complex what is elsewhere represented as simple. But through simple symbols alone objects cannot be represented as of different forms.

When symbols or objects are grouped, the groups will be formally alike or different in several ways: in *multiplicity*, in *type*, and in the distribution of identical elements (or tautologous symbols) within the constituent groups. (These terms are arbitrarily chosen to suggest the concepts they mean.)

The *multiplicity* of a group is determined by the number of major members.

However simple or complex the elements of a fact may be, these elements, as they are seen through a symbolic expression, are of a definite number; and the elements of the fact are correlated with the constituents of the symbolic group so that for each symbol, simple or complex, in the expression there is a corresponding part of the fact, and so that the expression as a whole means the fact as a whole. The first layer of facts will be those represented by symbolic groups whose elements are all simple symbols. "The sunshine on the ocean" is such a fact; $(2 + 3)$ is another such fact. Clearly, neither of these symbolic expressions could mean a fact which could not be analyzed into at least three elements. The major members of these facts (but only of this simplest type of facts) are the unanalyzed objects signified by the simple symbols. The number of these objects determines the multiplicity of the fact, and the number of simple symbols, the multiplicity of the symbolic group.[1] But the major members are not always simple symbols or the objects meant by simple symbols; the major members may be complex, and the multiplicity of the expression or the fact will not then be fixed by the number of simple components.

Primary facts, facts of the lowest layer, may enter as wholes with other constituents to form new facts; and these will belong to a second layer. At least some of their constituents will be

[1] The article, "the," does not represent a constituent of the fact. With it must be classed "a," "any," "some," "every," "each," and "all." See below, ch. IV, sec. v. Similarly, "not" does not stand for a constituent of a fact. See below, ch. VI.

themselves complex, and will be signified by groups within groups. This gives the second dimension of logical form — the *type* of the group. Horizontally, so to speak, a form is determined by the number of its major members; vertically, by the number of groups included within groups.

The sentence, "the sunshine on the ocean pleases me," represents a fact whose constituents are not unanalyzed objects signified by simple symbols. One of the constituents of this fact is itself an analyzed fact reproduced by a symbolic group. The whole includes a subordinate group, "the sunshine on the ocean," and this is combined with the things meant by "pleases" and "me" to form a fact of higher type. The secondary fact therefore contains a primary fact as one of its constituents.

Now the sentence, "I am happy because the sunshine on the ocean pleases me," means a fact of a still higher type. The secondary fact, "the sunshine on the ocean," has become an element in a new whole through its union with the things meant by "I," and by "happy," and "because." (The copula, "is" and its variants, is a sign of assertion; its function is unlike that of any other sign.)[1]

Facts combine and recombine in this way, reaching higher and higher levels of complexity. The result is a multitude of new logical forms — new combinations of symbols with symbols and objects with objects.

The *major constituents* of such facts and symbolic groups are the facts and groups which enter immediately into the whole without being constituents of sub-facts or sub-groups. All the other elements are minor constituents. The fact, "I am happy because the sunshine on the ocean pleases me," contains as major constituents the two subordinate facts, "I am happy" and "the sunshine on the ocean pleases me," and these are joined by the major relation "because." These subordinate

[1] See below, ch. V, sec. xiii.

facts in turn contain their major constituents, which are the minor constituents of the larger fact. The analysis proceeds as far as the facts composed of elements meant by the single words, and these facts contain only major constituents. Their elements are the basic materials of the structure.

The type of an expression or fact is therefore determined by the number of groups within groups. An expression or fact of the first type has no members which are groups. $(2 + 2)$, $(a - b)$, *France fears Germany*, are of the first type. An expression or fact of the second type will include as a major member at least one (or more than one) group of the first type; *e.g.* $(a\ (b - c))$, *France fears a new war*. The other major members of such a fact may, as in these cases, be things meant by simple symbols, or things meant by groups of the first type, *e.g.* $((a + b) - (c + d))$. In general, an expression or fact of the nth type will include as a major member at least one (though it may include more than one) group of the $n - 1$st type; and as a consequence, it will include as minor members groups down to the 1st type.

In determining the multiplicity of a group, its type must be taken into consideration, for it is the number of *major* constituents (the members entering immediately into the whole without being constituents of sub-groups) which establish the multiplicity, and these may be of a higher type than the first. Thus $((a - b) - (b - c))$ has only three major members, that is, the operation $(-)$, and the complex terms $(a - b)$ and $(b - c)$. Its multiplicity is not determined by the number of simple symbols or unanalyzed elements.

If facts of a secondary, tertiary, or higher type are alike both in their horizontal and vertical dimensions, they will be of the same form. This means that they must have the same number of major constituents, and that each of these constituents in the one must be constructed of groups within groups exactly as is a corresponding constituent in the other, and vice versa.

V

Language exhibits the form of the facts it signifies much less clearly than do some other kinds of expressions. This is because the grouping of words into phrases and sentences is brought about by grammatical rules, which are felt rather than made evident in the group. Nevertheless, every phrase or sentence of any complexity is organized into groups within groups, as is also the fact for which it might stand. The major verb or connective shows the major lines of cleavage; it connects the major symbolic members. These in their turn are composed of minor groups, joined by minor verbs and connectives. Punctuation plays a part here; commas, semicolons, dashes, brackets, periods, etc., indicate what symbols are to be taken together. These are signs of syntax.

Signs of syntax are secondary signs — signs of symbolism rather than signs of objects. They do not refer to elements in the fact for which the symbolic group might stand. They refer merely to the symbols. They mean that the symbols with which they are placed are to be treated as a group; that these symbols constitute a significant whole.

Mere juxtaposition of symbols, although it is usually a sign that they are a group — and is the only sign of syntax in the system of mental images — is not a sufficient sign in most systems. There must be additional ways of indicating the grouping. In language there are at least four signs of syntax: classification of words and expressions as parts of speech; inflections, case endings, genders, etc.; position; and punctuation. The signs of syntax in algebra, arithmetic, and all quasi-mathematical groups, having been invented for convenience, are simpler than those of language. The form of an algebraic expression is indicated by parentheses; thus $((a + b) \times c)$ is plainly similar in grouping to $((d - e) \times f)$. These expressions are of the same logical form — a form which can be described as a group of

three major members, one of which is a group of three unanalyzed major members. The parentheses are the signs of syntax, that is, the punctuation of these mathematical expressions.

In Messrs. Whitehead and Russell's *Principia Mathematica*, the signs of syntax are dots; the larger numbers of dots indicate the more inclusive groups, the smaller numbers, the minor groups. Any signs of syntax must be such that they preserve the integrity of the groups and show what groups are members of other groups.

As the forms of facts and the symbols which represent them grow more complex, verbal descriptions become more lengthy. There is a simpler manner of exhibiting forms. If letters are taken to stand for the elements of a fact (or a symbolic group) and parentheses as signs of grouping, any logical form can be presented in a general schematism. $((a\ R\ b)\ S\ c)$ exhibits the form of the two algebraic expressions given above, $((a + b) \times c)$ and $((d - e) \times f)$, and of the facts for which they might stand. It also exhibits the form of the sentence, "Brutus was the friend of Caesar," and of the fact which this signifies. This sentence, and the fact, fall into major and minor groups as follows: (Brutus was (the friend of Caesar)), and except for spatial order, which can be disregarded here, the symbols are grouped exactly as is the expression $((a\ R\ b)\ S\ c)$.

The use in this general schematism of capital letters for some of the group-elements should be noted. Wherever there is a combination of elements, the terms combined must be distinguished from the relation or operation which unites the terms. Qualities, relations, and operations enter into, modify, and give unity to groups of terms in a peculiar way, and they can be classed together as *elements of unity*, which are distinguishable from the terms they unify. A quality is a modifier (or element of unity) in a group of one term, *e.g.*, "the white knight," which is of the form (Ra). A relation appears in a group of several terms, *e.g.*,

"*a* between *b* and *c*," and an operation may enter in a group of any number of terms. The capital letters in this general representation of logical forms stand for symbols or elements of unity, while the small letters stand for terms.

Similarity of form between groups, whether it be between facts, between symbols, or between symbols and facts, can be shown by correlating them as follows: Each major constituent of the one must be made to correspond with one and only one major constituent of the other, and vice versa; and if these constituents are groups, each of their major members must be made to correspond uniquely, and so on, down to the elements signified by the simple symbols. This correlation will give a perfect parallelism of members, both horizontally and vertically. (Possible complexities in the element or symbol of unity are neglected. There will be at least one element or symbol of unity in every group, and although this may be complex, it can be treated as one — as *the* element of unity.) [1]

When a symbolic group stands for a fact, such a correlation of the group with the fact is set up by the relation of meaning.

This parallelism of groups, of terms and elements of unity, of complex and simple constituents, makes possible a comparison of the structure of groups without counting the number of the members or the layers of superimposed groups. We do not need the concept of the number-series in order to apprehend similarities and differences of form, but merely the concept of one-to-one correspondence, on which the number-series is based. The eye effects such a correlation in observing that ((*Rab*) *Sc*) and ((*Qde*) *Mf*) are of the same form.

VI

Symbolic groups thus constructed could not have a wholly arbitrary meaning; the principle of syntactical meaning makes it

[1] See below, ch. III, sec. xvii, for the additional differences of form introduced by the complexity of the element of unity.

necessary that in form at least they must reproduce the objects they might signify. Given significant simple symbols and a logical form, and the meaning of the whole will be determined by the meanings of the simple symbols and by the logical form. It will be possible to analyze the fact meant (if there is a fact) into groups and simple elements parallel to those of the symbolic expression.

Any fact can be analyzed in a number of different ways. Its lines of division and grouping run in different directions, depending on the elements chosen as the basis of the analysis. All the possible ways of stating the same fact will represent the different lines of structure, the different analyses. The fact, "*a* is between *b* and *c*," is the same (if *a*, *b*, and *c* mean the same things in both cases) as "*a* is to the right of *b* and *c* is to the right of *a*." "Caesar conquered Gaul" and "the first Emperor of Rome triumphed over the region which is modern France and Germany," are different analyses of the same fact. No fact and no object is exhausted by a single analysis. The analysis directs our attention to certain aspects of the fact, but other aspects of it remain untouched. There is no single logical form which is peculiarly the form of any one fact.

Statements of the type, "*x* is *y*," where "is" means "is identical with," tell us that a single fact permits different analyses; a large part of our knowledge is embodied in observations of differences of structure in the same fact. "*2* + *3* is *5*," for example, informs us that the object which we signify by "*5*" can be decomposed into the constituents meant by "*2*" and "*3*" and "*plus*." And if statements of identity were trivial, as they are sometimes said to be, much of our knowledge would be trivial. It is doubtless of no interest, excepting as a case of a general logical principle, to know that "*a* is *a*," and only of slight interest to know that "*a* is *b*"; but it is of great interest to know that the same fact can be stated in different ways, that a single object exhibits a variety of logical forms.

VII

We have yet to consider the third element of form — the distribution of identical members, or tautologous symbols, among the constituent groups of a fact or a symbolic expression.

Meanings must have a certain fixity and continuity. If symbols were fluid and capricious in their references they would be of no use, for we should never know exactly to what they referred. In any discourse symbols recur, they appear again and again, and on each recurrence they are given the same interpretation.

Now, strictly speaking, a repeated symbol is not the *same* object. The sound "bird" uttered to-day is not identically the sound "bird" uttered to-morrow. The image of my friend which is in my mind now is not identical with the image of him which was in my mind an hour ago. Therefore a symbol is something more than a single object which comes before the mind, takes on meaning, and disappears. Recurrence is of the essence of symbols. The same word may enter many times in one sentence and many more times in a paragraph. A number or letter may reappear in different parts of a single mathematical or logical expression. We cannot say that the word, the number, or the letter is the very same in each instance, but all the instances are like enough to be classified together, though each is a distinct object. The symbol is all the cases of its own recurrence.

A symbol then is not a single object, but a class of objects. Symbols are the same symbol only if they exhibit a generic similarity as objects, and they are distinct symbols if they exhibit a generic difference. Thus *a* and *b* are distinct symbols because their contours as marks on a page are different, while *a* and *a* (if they are not ambiguous) are the same symbol because their contours are alike. When we select an object as a symbol, we fasten on one or more of its concrete characters, and only objects with these characters are treated as the *same* symbol, while objects

of different characters are treated as distinct symbols. The characters that give generic similarity or difference to symbols may be any characters whatsoever. In European languages, for example, differences of pitch do not differentiate spoken signs, while in Chinese they do.

Apart from ambiguity, which must be dealt with as a special factor, a recurrent symbol always means the same object. Recurrent symbols are tautologous; they are not symbolically distinct and yet they are not identical as objects. But when it is said that tautologous symbols mean the same object, this "sameness" is not the sameness of *similarity*. The object is identically the same. Identity is that which makes an object one and unique. No two objects which are alike can be identical, for they are two and must therefore be at least numerically distinct. The recurrent instances of a symbol, though they are alike, do not signify objects which are merely alike. They mean an identical object.

The principle which gives rigidity to the meanings of symbols, that is, that *the same symbol, if it stands for an object, stands for an identical object,* is the principle of identity. Not only is a symbol a class of objects; it is a class of objects whose meaning is unique. Symbols which have different meanings are different symbols; it is impossible that the same symbol should have more than one meaning.

The attempt to interpret a symbol in more than one way gives rise to psychological ambiguity or equivocation; and this must be distinguished from the logical ambiguity of a phrase such as "a man" or "some man." Logical ambiguity is useful in thought, but equivocation makes precision of conception or statement impossible.[1] Equivocation is eliminated only when

[1] It is true, however, of logically ambiguous symbols, as of all symbols, that they follow the principle of identity. They can have only one meaning. The variable, "a man," if it is given a value, can be given one and only one value.

the equivocal symbols are treated as distinct. Though the spoken words "read" and "red" do not differ in their character as objects, they are different symbols in the sentence, "I read a red book," and they cannot be construed as the same without a distortion of the meaning. Needless to say, images are even more open to ambiguous interpretations than words.

Every symbol, then, presupposes the uniqueness of its own meaning, and this is what the principle of identity, *a is a*, tells us. It states that *a*, wherever it occurs, is to be interpreted by one and only one object; and if *a* is significant, but stands for no object, the principle tells us that *if a* represented an object it could be one and only one.

This principle is therefore a general rule of symbolism. Phrases such as "the round square" and names such as "Phoebus Apollo," which refer to no existent objects, follow the law of identity no less than do phrases such as "the present king of England" or names such as "Julius Caesar." "A round square" *is* "a round square," "Phoebus Apollo" *is* "Phoebus Apollo," in the sense that each of these symbols is to be interpreted in one and only one way, in the sense that each has a meaning of its own different from that of all other symbols not expressly defined to have the same meaning. "Round square" plays the same part in the symbolism, wherever it appears, as "round square"; instances of this symbol can replace one another in any expression without altering the meaning of the whole. In short, as a general rule of symbolism, *a is a*, states that *a* is the same symbol as *a*. It declares an intention to interpret *a* uniquely. The very conception of a symbol as distinct from other symbols and the same in all its instances demands that its meaning be unique.

The principle of identity can also be interpreted in another way, that is, as a statement of fact about objects; but this second interpretation presupposes the first—the interpretation as

a rule of symbolism. When *a* stands for an object, *a is a* asserts, not merely that *a* and *a* are the same symbol, but that the object meant has an identity, that it is an object. This is the existential as distinguished from the symbolic interpretation of the principle of identity. And if *a* is a variable, so that it means any object, *a is a* affirms that "any object is identical with itself."

Clearly, such a statement of identity is false if it refers to no object; it is not true that "a round square is a round square" in the sense that round squares have identity. However, "*a is a*," "Caesar *is* Caesar," "man *is* man," could not be asserted as existential propositions unless "*a*," "Caesar," "man," were construed as the same symbols in each case. It must be assumed that the meaning of a symbol is unique if any object is affirmed to be identical with itself; but it is not necessary to assume that the symbol stands for an object in order to assert that it can have one and only one meaning, for this latter rule is a condition of its use, whether or not it stands for an object.

Distinct symbols, symbols whose characters as objects are dissimilar, may be equivalent in meaning; they may stand for an identical object, or if they stand for no object, they may be treated as indistinct in meaning. This treatment will not make them the same symbol, for symbols must be alike as objects to be the same. Thus "red" and "vermilion" are distinct symbols with the same meaning; but they are not tautologous or recurrent instances of one symbol. Such equivalences may be equivalences by definition only, and they will then be statements of special intentions in the use of symbols: the equivalence "*a* is *b*" tells us that *a* and *b* can be substituted for one another wherever each occurs. This equivalence is a valid definition, but as an assertion of fact it may be false. Thus "Titania" is "the queen of the fairies"; but there are no fairies and no Titania. Definitions do not require the existence of an object defined, for

they are primarily concerned with the use of symbols, and a valid definition may be false as a statement of fact.

VIII

The proposition, *a is a*, when it means that any object is identical with itself, is true without exception. There is nothing of which we can speak that is not self-identical.

The concept of the identity of an object is said by Hume to be a fiction. Objects weave themselves in and out of experience as do threads in a tapestry, and it is difficult, if not impossible, to know that a thing which appears the same is really the same. But the entire fabric of thought would fall apart if we did not assume that an object fixed as a thing meant can remain identical with itself.

Putting aside the question of the reality of these self-identical objects, we must suppose that objects — for knowledge at least — stand still long enough to be referred to and re-referred to, for if this were not so nothing could be represented. All objects, including symbols themselves, would flow together; there would be no differentiation in knowledge. Nothing would remain itself to be an instrument of significance or a thing signified. We could not relocate anything we had previously known, and we should be helplessly blown about by the winds of change. Thought must view the world as if it were made up of points and lines of identity, which combine with and intersect other points and lines, but which do not cease to be themselves.

The nearest approximation to a definition of an object (though it is not a definition) is this: that an object is anything with the twin characters of identity and diversity; and this statement is equally true of subjects, predicates, terms, relations, and complex wholes. A universal such as *whiteness* has its identity; it is *something* because it is identical with itself and distinct from other things — from other universals and from each

of its own instances. A particular, referred to by "this white object," for instance, is more obviously self-identical and diverse from other objects. These two formal characters are the sources of unity and plurality in the world. Wherever there is an object it must be a unity — one and identical with itself — and there must be other such unities from which it is distinct.

It would be natural to suppose that the identity and diversity of objects arise through special qualities or predicates, or through sets of relations, which attach to objects. But if we attempted to carry this idea forward, we should then have to explain why these predicates or sets of relations are unique, why they are themselves and not something else; and we should have made no advance. Identity and diversity are irreducible concepts, and the argument which shows most convincingly that they are indefinable is that the characters and relations in terms of which they could be defined (if at all) must themselves be assumed to be self-identical and diverse from all other characters and relations.

When symbols copy the forms of objects, they reproduce the identities and diversities among objects, together with the grouping of these identical and diverse elements into wholes. A symbolic group is a black and white portrait of a fact; it leaves out everything but the identities, diversities, and groupings of the original. But the representation of the identities is a projection, rather than a copy; for an identical element, entering and reëntering in several places in a fact, is signified not by an identical symbol but by a recurrent symbol. The identical element is projected into the medium of time (or space) in the representation. What is a single thing in the fact is reproduced by a series of similar things in the significant expression, that is, by a number of instances of the same symbol, and the convention by which this projection is effected is the principle of identity. For this principle affirms that if the instances of a symbol stand for

an object, this will be a single object. Tautologous symbols in the representation stand for identities in the fact.

An identical element may enter in many different ways in a fact. Many different complexes may be centered about the same object to constitute a whole, and this identical component need not be recurrent in a spatial or temporal sense. "Hamlet's father was Hamlet's Nemesis," states a fact in which the same object enters in two different ways: the whole fact is composed of two major groups with an identical constituent in each. It is of the form $((a\ S\ b)\ Q\ (a\ S\ c))$. Such a fact is like a number of intersecting lines; it is made up of groups with common members.

Still other facts are reflexive they; arise through relations which hold between a term and itself. The identical element in a reflexive fact enters as a major constituent at least twice in the same group. "Hamlet feared himself" stands for a reflexive fact, for the fact "Hamlet feared Hamlet," which is of the form $(a\ R\ a)$. Strictly speaking, we ought not to say in such a case that the fact is reflexive; rather, it is the symbolic expression which is reflexive, for here again the expression does not exactly copy the fact. The fact has only one term, while the symbolic group has two, though these two are not distinct. The fact is really of the form (Sa); and it is through the tautologous recurrence of a symbol that we can represent it as of the form $(a\ R\ a)$. Thus what is in fact a quality (for a quality is an element of unity attached to a single term) is symbolized as a relation. "Napoleon loved Napoleon" means the same thing as "Napoleon was egotistical"; and the possibility of representing an identical term through a recurrent symbol enables us to treat this quality as a relation — a relation between a term and itself.

In determining the multiplicity of such reflexive expressions, the different instances of the same symbol must be reckoned as separate terms or members. Thus $(a\ R\ a)$ has two terms and is the same in multiplicity as $(a\ R\ b)$. $(Qaab)$, which is the form of

"*a* gives *b* to *a*," where "gives to" is a single relation, has three terms and is the same in multiplicity as (*Qabc*), which is the form of "*a* gives *b* to *c*." It is clear that the multiplicity of the fact meant by these reflexive groups is not that of the symbolic group; a fact of a lesser multiplicity is represented, through tautologous symbols, as of a greater multiplicity.

The manner in which an identical object appears and reappears as a component in a single fact is the third distinctive feature of logical form; and facts or symbols of the same form must correspond not only in the number of their constituents, but also in the distribution of identical elements or tautologous symbols among the groups. Thus (*a R a*) and (*b S b*) will represent facts of the same form, which is that of "Napoleon loved Napoleon" or "Hamlet feared Hamlet." The symbols ((*a R b*) *S* (*a R c*)) and ((*e Q f*) *L* (*e Q g*)) will stand for facts of the same form — facts such as that signified by "Juliet's love caused Juliet's doom." It is easily seen that the appearance of an identical element in different ways in a fact leads to many new varieties of form.

These identical elements may also be represented by distinct but equivalent symbols; and here too the symbolic group does not exactly portray the fact, but contains more distinct members than there are distinct elements in the original. "Hamlet mistrusted himself" is composed of three distinct symbolic members, ("Hamlet" and "himself" are symbolically distinct), while the fact to which this expression refers contains only two. The expression is redundant in form. Such a group can always be reduced to a more exact copy, or projection, of the form of the fact by replacing the equivalent symbols by instances of the same symbol; and this is what is done in interpreting the expression. Its meaning is the same as that of "Hamlet mistrusted Hamlet."

The distribution of tautologous symbols or identical elements

in the constituent groups of a symbolic expression or a fact takes a place of equal prominence with multiplicity and type in the determination of logical forms. For the distinction between a reflexive and a non-reflexive group, or between a group whose sub-groups contain common members and one whose sub-groups contain no common members (or differently distributed ones), is a formal and not a material distinction.

IX

Among the more important differences of form which have not been mentioned are those of order — the difference between "*a* precedes *b*" and "*b* precedes *a*." Order requires no new concepts; it is determined by grouping alone.[1]

In the present chapter we have shown how logical form arises from the grouping of self-identical and diverse objects into facts, and of tautologous and distinct symbols into significant wholes. The structure of a symbolic expression copies the structure of the fact which it means, or might mean, through its grouping.

The three chief features of logical form are: (1) multiplicity, (2) type, and (3) the distribution of tautologous symbols or identical elements within groups. The multiplicity of a group is determined by the number of major constituents, recurrent symbols being counted as separate constituents; the type by the number of groups within groups down to the elementary constituents. The major members of any group are those which enter immediately into the whole and are not members of minor groups. These major members may be simple or complex.

The distribution of identical elements in a fact is not exactly reproduced in the symbolic group; these identical elements, which may not themselves be recurrent, are represented by recurrent symbols. Groups of like form must correspond with one another in this respect, as well as in multiplicity and type, and in

[1] See below, ch. III, sec. xiv.

the multiplicities and types of their sub-groups, down to the simple constituents. The principle which permits us to represent identical elements in a fact by recurrent or tautologous symbols is the principle of identity. A symbol is not a single object but a class of like objects, and in any or all of its instances its meaning is unique. The principle of identity tells us that a symbol can have one and only one meaning; and this is the case whether or not the symbol stands for an object. The principle of identity is therefore primarily a general rule of symbolism, but it can also be interpreted as stating a fact — the fact that "any object is identical with itself." Its use in this sense presupposes its validity as a rule of symbolism; but a statement of identity which is false as an affirmation of the existence of a self-identical object is valid as an affirmation that the same symbol is always used in the same sense. Distinct symbols may be defined as equivalent in meaning, regardless of their reference to existent objects. Identity and diversity are indefinable.

Equivocally interpreted symbols are distinct, for a single symbol can (by the principle of identity) have one and only one meaning.

Simple symbols are not distinguished from one another by their form, for they are not groups. They represent objects with a zero analysis; they set the scale of reproduction; but the objects which they represent as simple may be in reality complex. These objects are taken as simple merely for the purposes of representation. Any single fact permits many alternative analyses; it may have many different forms; and no analysis exhausts a fact.

The concept of a group, on which the notion of structure rests, is the same as the concept of function. A group is (1) a plurality of objects and (2) a whole determined by its parts, and the relations between them; and (3) in the case of symbolic groups, the meaning of the whole is determined by the meanings of the

parts. Signs of syntax—punctuation, parentheses, etc.—indicate what symbols are to be taken together as significant groups. Logical forms can be represented in a general schematism through letters grouped in parentheses. Thus (*Rab*) and (*Sabc*) differ in multiplicity but not in type; ((*Sab*) *Q* (*Ldf*)) and (*Rab*) differ in type but not in multiplicity; ((*Sab*) *Q* (*Ldf*)) and (*Sabc*) differ both in multiplicity and in type; and ((*Raab*) *Sa*) and ((*Qabc*) *Sd*), though they are alike in every other respect, differ in the distribution of identical (or tautologous) elements. Comparisons of form, since form is numerical in nature, can be effected by a one-to-one correlation of groups without the use of the concept of the number-series.

The capital letters in this schematism signify elements which perform a unique function in each group, elements of unity, that is, relations, operations, qualities; and these must be distinguished from the terms related, qualified, or operated on.

Logical form is so woven into speech, and even into the play of the imagination, that it is impossible to utter a phrase or call to mind the images of a past or future experience without throwing them into the forms we have described. To say that a fact is not of logical form is to say that no significant assertion can be made about it; it is to say that it is not a fact. Such a "fact" is placed beyond the reach of thought or rational experience.

CHAPTER III

UNIVERSALS AND INDIVIDUALS: ORDER

I

EVERY presented object has two aspects: it is both universal and individual, a "what" and a "this," an essence and that in which an essence is embodied; and we have not completely described logical form until we have considered this cleavage of the objects of knowledge into universals and individuals.

Objects are not perceived without their natures and their relations to other objects being also perceived. Each is a "this" of some quality, or a "this" related to something which is diverse from it; and the minimal cognition of objects — a perception of the distinctness of "this" and "that" — contains a universal, for *distinctness* is a universal. If one attempts to grasp the "this" without its qualities or relations, it is no longer a "this"; it becomes so vague as not to be known even as "something-or-other." We lose it in the pure awareness where there is neither "this" nor "that." And yet no object is perceived as a pure quality or relation. I am not presented with sheer *whiteness* or *betweenness* or *beforeness*. A perceived universal is perceived in something, as qualifying a subject or as relating terms; and this subject or these terms are not themselves merely qualities or relations. There is something in perception to which the universal is united. The universal is perceived as individualized.

No distinction has given rise to wider divergence of opinion. Plato dismisses the imponderable something in perception which is not universal as an element of imperfection; but this indeterminate something reappears as the unintelligible "matter" or absolute non-being of his cosmology. In Aristotle's metaphysics

the same imponderable, far from taking the form of absolute non-being, becomes infinite potentiality, infinite possibility of being; but Aristotle finds no actual being which is not also universal. The nominalist, on the one hand, affirms that only the individual is real; and the realist — the extreme realist — that only the universal is real. But if with the Platonic realists we turn our minds away from the things of perception and deny the reality of individuals, we are still confronted with the facts of perception; and though these facts are proclaimed to be "opinion" or "illusion," the duality of universal and individual, in a very persistent form, continues in knowledge. Nor does the denial of the reality of universals make them any the less important as phenomena of knowledge.

The distinction between the "what" and the "this" is omnipresent in knowledge whether or not it is in reality.[1]

II

One of the paradoxes of knowledge is that having made the distinction we attempt to escape from it by describing the individual in universal terms.

My house is built of brick and stands on a hill. I return to it after a day's absence to find that the red walls still top the hill; but what I recognize are the characters and relations, and not the individual thing—my house. If I try to describe its individuality, I find that this dissolves into new characters and relations; it continually recedes from my grasp. I have embarked on an apparently endless process of distinction and characterization. Predicates split off from the individual and leave it intact. The very name of an individual takes on universal significance;

[1] This distinction between the universal and the individual is not the same as the distinction between *essence* and *existence*, which Mr. F. H. Bradley describes as the separation of the "what" from the "that." (See F. H. Bradley, *Appearance and Reality* (1893), ch. xv.) An individual, a "this," may or may not be in existence, yet its individuality, its "this," will be distinct from its nature, its "what."

we speak of "a Solon" or "a Solomon." From "Caesar" is derived "Caesarian"; and we refer to "Caesarian pomp" or "Caesarian power." What was "Caesarian" in Caesar was not then the individual himself, but a quality of the individual. A distinction of the "what" from the "this" always leaves a "this" in which a new "what" can be distinguished.

The problem of whether the individual is completely describable in universal terms is one form of the general problem of determinism. Laws are descriptions of phenomena; and determinism in one sense means that all phenomena follow laws — that they can be described in universal terms. And if the task of knowledge is completely to describe its objects through laws — to reach a "specific essence" of everything — then knowledge defeats its own purpose by admitting the distinction between universals and individuals, unless it can finally abolish the distinction and identify an individual by its predicates and relations.

The pursuit of the "specific essences" of individuals makes it equally plausible to assume either that individuals are not identified by their predicates and relations, or that they are. The question raises a genuine antinomy. The fact that every individual can be more and more fully characterized, that the margin of confusion with other individuals is narrowed as more and more predicates are affirmed of it, seems to be evidence that a sufficient number of predicates — perhaps an infinity — would exhaust the individual; but it is equally good evidence that no predicates exhaust it. If I say of the tree beside my window that it is a pine with bark of grayish green, that it stands six feet from the house and thirty feet high, that its branches are covered with moss—if I describe it by these and a thousand other predicates, it is still *that to which* the predicates attach, and I can find more predicates. The universals are like intersecting lines at a focus, excepting that the lines determine a point, while the in-

dividual seems to lie outside all intersecting universals; and there is no contradiction in assuming that another individual might have all the same predicates and relations.

III

But is an individual not determined by its spatial and temporal relations? Is it not impossible that two individuals should be in the same place at the same time? And if we can know *where* and *when* an individual is, have we not described it beyond all confusion with other individuals?

There is in perception a direct apprehension of the numerical diversity of individuals — of the "this" of one object and the "that" of another — and this perception, taken together with the perception of change, is the basis of the concepts of space and time. This is the sole manner in which an individual is known as distinct from other individuals. Of two peas which are exactly alike in every respect the only difference I can perceive is a bare numerical diversity, which is not a diversity of characters and relations, but a pure difference of individuality. To be thus perceived as distinct, individuals must be copresent, and they must neither change nor be in motion. The "this" and the "that" must form a changeless whole of perception. This numerical diversity may, moreover, be given as a relation between several terms; the distinctness may be one of three or four, or possibly half a dozen, individuals.

If, along with this perception of numerical diversity, goes a perception of change, the phenomenon is ambiguous; there is a question as to whether a "this" which is present at a later stage of the change is the same as the "this" of an earlier stage. The two individuals are not perceived as distinct, since they are not copresent; and yet they are not perceived as the same. Imagine a uniform field of gray before the eyes and suppose that this becomes green and then red. Is the "this" of the object the same

beneath its changing aspects, or is each perceptible difference of quality joined to a different "this"? Is the individual momentary, or does it persist through time?

Change, introducing as it does the element of time, is open to two interpretations: it may be that the qualities and relations alone are different, while the individual remains the same; or that both are different. The change may be like the flow of a river, a continuous displacement of one individual by another, or like the approach of two moving bodies, an alteration of qualities and relations without the entrance of new individuals. From one point of view it seems that in each moment of time individuals are destroyed and replaced by new individuals, while from another point of view it seems that the individual must persist, at least for a brief period of time. How are we to recognize either the destruction or the persistence of an individual? How are we to know that the "this" of one situation is the same as the "this" of another? We can interpret our memory of the historical continuity of our own selves as the perception of an individual persisting through time; but if this is what we mean by the perception of a persisting individual, certainly we know only one individual in this way. Whenever we believe that we are perceiving in the outer world an historically continuous individual, whose qualities and relations alone are changing, it is always possible that Descartes's playful demon is at work, substituting distinct individuals along with the different qualities and relations; and if a changing object is viewed at remote times, it becomes increasingly difficult to believe that the individual persists through change.

Individual objects can be perceived to be distinct and self-identical only if they are held together in a single whole of perception. But this does not amount to the scholastic principle of individuation — that the place of an individual at a certain time determines its individuality. This principle reverses the

true order of the concepts of spatio-temporal position and individuality. The reason why individuals cannot be in the same place at the same time is that their numerical diversity and self-identity as individuals — apprehended in a single whole of perception — gives us the meaning of "the same or different places at the same time."

If there were any such thing as an independent space-time, it would probably be true that position in space and time would identify individuals. But it is impossible to view space and time as independent entities with reference to which objects take positions. Space and time are functions of objects; there are "places" in space and "moments" in time because individuals are self-identical and distinct. If I wish to show where and when an individual is, I must do so by referring it to other individuals from which it is numerically diverse, and so on, *ad infinitum*. The spatio-temporal relations of individuals are not describable without the use of other individuals as a frame of reference, and since the individuals which constitute this frame of reference are not themselves identifiable by their spatial and temporal coördinates, no individuals are identifiable in this way. Space and time are not peculiarly privileged universals. Like all other universals, they appear only as individualized; and spatial and temporal predicates have no more claim to define the individuality of a "this" with which they appear than do other predicates.

IV

To solve the antinomy of the determination of the individual by the universal, Leibniz makes use of the postulate of "the identity of indiscernibles": no individual can share all its predicates and relations with another, that is, only that which is distinguishable by predicates is distinct. But this postulate takes advantage of the weakness of our powers of knowledge; it may be that we fail to make distinctions where distinctions exist.

The postulate needs to be reënforced by another, "the principle of sufficient reason," from which it follows. This principle asserts that everything has a reason, and since reasons are universal, that everything (including the individual) is describable in universal terms. Thus the individual is connected logically, deductively, with the universal.

But if an individual is determined by its predicates, so that no individuals can have all their characters and relations in common, the process of distinguishing characters and relations in individuals should sometime come to an end. The "specific essence" of an individual should sometime be reached, unless it takes an infinity of universals to determine an individual. Faced with the apparently endless exfoliation of the "what" from the "this," Leibniz added that the number of universals which determines an individual is infinite; and this leaves finite knowledge no better off, so far as the determination of individuals by universals is concerned, than if the individual were not so determined.

If the assumption of the identity of indiscernibles is true for reality, it is nevertheless useless for finite knowledge. The distinction between the universal and the individual persists in its original form for perception and thought. The metaphysical postulate leaves the epistemological difficulty as it was.

V

It is possible for knowledge to proceed — and it does as a matter of fact proceed — without answering the question as to whether the individual is describable in terms of universals. The assumption of determinism (or indeterminism), even in this most abstract form, is not essential to thought.

It may or may not be true that a sufficient number of predicates determines the unique "this" of an object; the individual can still be thought of, it can still be referred to through sym-

bols. The individual appears in conceptual knowledge as an x, as something represented by a variable symbol which we assume to have a meaning, though what this meaning is we cannot say. We cannot even assert that this x might not stand for an aggregation of universals, for it is always possible that Leibniz's infinity of predicates might be its value. But if we could not employ variables, which are symbols whose meaning is undetermined, we could not bring the individual into thought.[1] Thus knowledge of the individual always takes the form (xR), where x is a variable term and R is a quality of that term, or the form $(xy \ldots S)$, where x, y, etc. are variable terms and S is a relation between them. (S or R might also be a mathematical or logical operation.) If one asks what it is to which the relation or quality attaches, he is driven to descriptions in terms of other qualities and relations; the term is lost behind essences which are *of* it but which are never it. In short, the value of the variable continues undetermined.

The representation of an individual through a variable eliminates a possible confusion between the individuality of a thing and its reality or existence; it makes clear the point that individuality may be purely conceptual. If the "this" of an object — the ultimate subject of all its predicates or the final term of all its relations — is simply the real or the existent, the individual is destroyed; it is absorbed in reality. This ultimate subject or term is represented by a variable as distinct from other subjects or terms — as one of a plurality. The x which gives individuality to one essence is thought of as other than the y which gives individuality to another; the indeterminateness of the x or y is the counterpart in thought of the inexhaustibility of the individual. Thus things of the imagination, which have not been presented and which may or may not exist, are individualized. Witness the characters of one's favorite novel who, if they live in the

[1] See below, ch. IV, sec. iv, for a general discussion of the variable.

mind, are as inaccessible to complete description as any real persons.

It might be thought that a proper name stands definitely for an individual, that the variability which is present in the representation of individuals is thus eliminated. But if proper names are more closely examined, they are found to be subject to the same indeterminateness which infects all references to individuals. The name "Napoleon" or the name "Stratford-on-Avon" stands for one and only one individual, but *which* individual we can know only through description (or else as a mere "this" numerically diverse from a "that"). Napoleon is "the conqueror of Europe," "the exile of St. Helena," "the defeated general of the Battle of Waterloo"; Stratford-on-Avon is "the birthplace of Shakespeare." Each is an x to which characters and relations attach, and the proper name no more determines what x this is than does the description; indeed, the meaning of the proper name is equivalent to the meaning of one or more of these descriptions of the individual named, and the variability present in the description is not absent in the name. The "this" of an object for which a proper name stands is as elusive as the "this" of any other thing.[1]

Since knowledge presupposes the distinction between the universal and the individual, and since the individual is always (for knowledge) something other than its predicates and relations — represented in no other way than through a variable x to which these predicates and relations attach — knowledge presupposes its own indeterminateness. Leibniz's metaphysical principles of "the identity of indiscernibles" and of "sufficient reason" postulate that this indeterminateness would disappear in an infinite knowledge; such a knowledge could assign a value to the x which represents the individual. But for finite thought, only propositions which make no reference to the individual can be free from

[1] See below, ch. IV, sec. ix, for a discussion of proper names.

this indeterminateness — only propositions which are stated in universal terms can have an invariable meaning. For we can know what we mean by a symbol which stands for a universal, but we can never know what we mean by a symbol which stands for an individual; or rather, we can know only that we mean a "this" which is distinct from a "that."

VI

The universal is determinately known, it can be represented in thought by a symbol whose meaning is invariable, because it is presented as self-identical in different "thises" and "thats," in different individuals. And when change renders the identity of the individual ambiguous, the universal — or at least some universal — remains what it was. Universals are strands of identity which spread themselves out through time and space; it is of the essence of universals to be recognized as the same in changing and diverse instances.

There is also a perception of the distinctness of universals, which is perhaps more primitive than the perception of their identity in different instances. Cutting across the numerical diversity of individuals, when they are copresent as "this" and "that," is a second (numerical) diversity — that of qualities and relations. The "this" is apprehended as distinct from the "that" in some other respect than its individuality. It is a "this of the sort *X*" as distinguished from a "that of the sort *Y*." And if there are several "thises" and "thats," two or more of them taken together may be distinct in some other respect than individuality from two or more others taken together. This will be a perceived difference of relations, *e.g.*, if four circles are presented, two including one another and two excluding one another, the inclusion and exclusion are aspects in which the *groups of two* differ. "Inclusion" is the "what" of one group, and "exclusion" the "what" of the other.

Such diversities of qualities and relations are *numerical*, no less than are those of individuals; the universals are "two" or more. But this second numerical diversity is additional to the first; it is a numerical diversity *in* numerical diversity. As with individuals, so with universals — they need to be copresent to be known as distinct. But despite the fact that the distinctness of universals is of the same general sort as the distinctness of individuals, the one is not reducible to the other, for the universal is always merely an aspect of the individual, the "what" and the "this" are always held apart. If I am presented with a white object and a black object, the perception contains two numerical diversities on different levels, the diversity of "this object" and "that object" and of "blackness" and "whiteness."

Along with this perception of differences of qualities and relations may go that of the identity of a quality or relation in different individuals. The "this" and the "that" may be perceived as the same in some aspect. When two white objects and a black object are given together there will be, beside the numerical diversity of the three individuals, a diversity of *two* qualities; but a single quality will be shared by two of these individuals. A self-identical universal will be recognized in different instances.

Mr. G. E. Moore discards this possibility of a universal being an "identity in diversity" because a self-identical object can be one and one only; and he believes, as a corollary of this, that a self-identical universal could not be split between several individuals and remain self-identical. The individual "whiteness" of one white object, he argues, is not identically the "whiteness" of another white object.[1]

The difficulty arises from a confusion of identity with in-

[1] G. E. Moore, "Identity," in *Proc. Arist. Soc.* (1900–1901), p. 103; especially pp. 115 ff.

dividuality. By a self-identical "whiteness" one does not mean an individual white; and if there is an individual white, this is not a universal but the same thing as is meant by "a white object." Now a white object is obviously not identical with another white object, but the "whiteness," as a universal, is the same in both cases. The fact that a universal occurs with individuals which are not identical does not rob it of its own identity. Since identity and individuality are not the same, a single identity can be individualized in many different cases. Each distinct quality or relation is something in itself; it has an identity which makes it what it is, and if it had none it would be a nonentity. But to say that a universal is an identity is not to say that it is an individual.

The perception of the numerical diversity of individuals, then, is accompanied by a perception of the diversity or identity of qualities and relations in these different individuals.

What of the perception of diverse universals in a *single* individual? Obviously, a white object can be both white and round; it can be included in or excluded from another object; universals, themselves static, can be grouped about a static "this," so long as no ambiguity concerning the identity of the individual is introduced by changes in the situation. Just as we perceive (in a single whole of experience) the self-same universal as qualifying or relating many different individuals, so we perceive the self-same individual as entering into many different relations and as qualified by many different predicates.

But every whole of experience gives way to new wholes; change is of the essence of experience.

The self-identity and distinctness of the individual slips away behind what seem to be its changing aspects, and the universal alone continues to be known as the same. The waxing and waning of a light is always light; the approach of two moving bodies, always motion; the alteration of colors in the field of vision, al-

ways color; the drops of water which displace one another in the flowing river are always of the same chemical composition. It is impossible to determine whether the individual persists, but a changeless universal is the background of the change; and the identity of this universal does not become ambiguous, as does that of the individual.

All perceptual distinctions, therefore, excepting an immediately apprehended difference of the "this" of one object and the "that" of another, are distinctions of universals; and all perceived identities, excepting the identity of a momentary "this," are identities of universals.

VII

What has been said so far of universals and individuals is preliminary to another point — to a further characterization of logical form. The idea of the structure of groups is not complete without the concept of a universal and of the part it plays in groups. From the point of view of logical form, universals and individuals are distinguished by their structural functions.

A universal is an aspect of a group as a whole. It is that which distinguishes the group from another whose constituents, apart from the universal, are indistinguishable from those of the former except by their bare numerical diversity. The group as a whole is an instance of this universal, which is itself an element in the group, but not an element coördinate with the others. The universal is an element of unity, while the others are terms.

Group-unity is the universal of universals, it is the "what" of every group; and different universals are different kinds of group-unities. To recognize anything as a group is to recognize it as a unity of a specific sort, that is, an instance of a quality, a relation, or an operation.

Consider the unity of a simple qualitative fact such as "Iago

was wicked." Here the predicate and the subject are knit together. The expression does not mean "Iago" and "wickedness" as a mere plurality, but "Iago-as-qualified-by-wickedness." And if the fact were a number of terms joined by a relation or a mathematical operation, the relation or operation would be attached to the terms in the same general manner as the quality to the thing qualified. The quality, relation, or operation is not related to its terms, for this would involve an infinite series of relations; we can say only that the terms enter into the peculiar unity which makes them a group or a fact. The fact "clouds precede rain," for example, is not made up of the elements, *clouds*, *rain*, and the relation *precedes* related to one another; for if this were so, there would need to be a third relation which unites the second relation to the original terms and relation, and so on. If the unity of a fact depended on the knowledge of such an endless series of relations, the fact would not be presented as a whole. The unity is given immediately. A relation relates its terms and a quality qualifies its subject once and for all; and this is true of the simplest fact — of a fact, for instance, constituted of the meagre relation of conjunction or the mathematical operation of addition. "*A* plus *B*" cannot mean *A* and *B* and *plus* added together, for this would demand an infinity of new *pluses*, and the fact would remain incomplete. *A plus B* is a new entity, something other than the entities *A* and *B* and *plus;* and it is given when the terms and the operation are given in the unity which makes the whole an instance of addition.

That a quality, when it modifies a subject, is *of* the subject in the sense in which an operation or relation is *of* its terms, is borne out by our habits of speech. Relations or operations are predicated of their terms as qualities are predicated of their subjects. One says "*x is* between *y* and *z*" as readily as "*x is* white"; *x*, *y*, and *z* as a group have betweenness as *x* alone has whiteness. Thus qualities, relations, and operations, despite their differ-

ences, are alike in their structural functions.[1] They can be distinguished from the terms they unite or the subjects they qualify, but at the same time they are cemented to these terms and subjects.

The ability to enter with other elements in a group so that the plurality of members becomes a whole is characteristic of universals; and each group contains a universal which performs this function in it. If a fact (a complex object) is presented, the determination of the group by the elements attests the presence of a universal. The form of a fact is a blank into which the constituents must fit, and at least one of the places in the blank is for a universal, while the others are for terms. And even the perception of bare numerical diversity, either of individuals or universals, is an analysis of a whole into parts which are diverse from one another but have unity through the relation of diversity.

A fact is therefore something more than a union of any elements whatsoever; it is a union of elements so that a universal enters into terms or subjects. The notion of a group involves not only the concept of members united into a whole, but also the concept of an element (itself a member) which determines the members to be a whole. But the universal is distinct from the terms because it fulfills a different office in the fact; it stands above the terms, pervading the whole and lending to it a distinc-

[1] Operations and relations, though they are like qualities in that they unite with terms in the same fashion, are not qualities. From the point of view of form, a quality modifies only one term; it is of the form (*Ra*). A relation has at least two terms; it is of the form (*Sab*); and if these terms are not distinct, as in (*Saa*), the relation still has two terms, so far as the representation goes (though these are not distinct), but the fact has only one. Such reflexive "relations" are qualities represented as relations between a term and itself. It is the symbolic group rather than the fact which is relational in form. Operations, on the other hand, may appear with one or more terms. They may be of the form of qualities or of the form of relations. But operational groups are distinguished from qualitative and relational groups by the way in which they are used in symbolic systems; they permit uses which these others do not permit. See below, ch. VII, sec. vii, for this distinction.

tive coloring. The concept of the unity of a universal and the terms on which it rests must be placed with the concepts of object, and of identity and diversity, among our indefinables. The Platonic description, "participation," as nearly characterizes this unity as any we know. If one were to revise Kant's list of the categories, leaving aside the question as to whether they are *of* the object or *of* the mind, he would find it necessary to include among the presuppositions of knowledge, not only the notion of the identity and diversity of objects, but also that of the peculiar unity of groups which springs from the presence of a "participating" universal.

VIII

If a universal is any object which can assume the rôle of element of unity in a group, an individual must be an object which can take no other part than that of a term.

Individuals alone do not constitute facts; there are no purely individual facts. Through universals we describe and relate individuals, but through individuals nothing is described or brought into relation. "The table," "the chair," "the book," cannot in themselves unite into a group; by their nature they are capable of playing the part of terms and nothing else; and only when they are related or qualified in some way do they become elements in a fact. (Indeed, an individual referred to as "a table," "a chair," or "a book," is already an x qualified by a predicate, and has already become an element in a fact.) "The table with the book on it and the chair beside it" is a fact; but "the table the chair the book" is no fact. Words and phrases that signify individuals do not, by themselves, go together to form significant expressions, and here again language reflects the structure of objects.

Without predicates, relations, or operations to bring them into groups, individuals (if they are thought of) continue in iso-

lation, cut off by their individuality from other objects. Such isolated individuals are never known concretely; it is only by the most violent act of abstraction that we can conceive of them.

The search for a relation between a universal and the individual which is an instance of it results, therefore, in the discovery that "being an instance of" is not a relation. The unity of a universal with individuals is a wholeness within which relations are distinguished; a relation is a species of group-unity; but this unity is not itself related to its terms, and the statement that it is "in" or "of" or "with" its terms serves only to point to it as an indefinable. The minimal object of knowledge is a universal in its instance; the universal, the individual, and their unity are given at once.

IX

The distinction between terms and elements of unity is not the same, however, as that between individuals and universals; for the terms as well as the element of unity in a fact may be universals, as in "truth is beauty," "dishonor is worse than death," "the course of true love never did run smooth."

A fact whose terms are universal is not the same kind of fact as one whose terms are individual. The latter is concrete; the former, abstract; but both are nevertheless facts. An abstract fact is an aspect of a concrete fact; or rather, it is an aspect of many such facts. But in perception, facts come to us as individualized, and not as abstract.

The ability of universals to fulfill a double function in groups — both as terms and elements of unity — divests them of the separateness which belongs by nature to individuals. In an abstract fact such as "dishonor is worse than death," a universal which might qualify or relate in some instance is referred to apart from its qualifying or relating function, and it becomes a term in a new fact; but the universal remains a universal be-

cause it can appear as a qualifying or relating element, rather than as something qualified or related. Thus when I say "this paper is white," I refer by the word "white" to the same object as that to which I refer by "whiteness" when I say "whiteness is the symbol of purity." The difference is not in the identity of the object meant, but in its function. In the one case it is a term in an abstract fact; in the other, an element of unity in a concrete fact.

We tend to speak of facts whose terms are individuals as "particular," but the word "particular" is misleading. Every object or fact is in a sense particular, for it has an identity of its own; it is a "somewhat" distinct from other objects, and it can be referred to as "this" object, "this" quality or relation, "this" fact. Particularity in the sense of identity is found everywhere; not only in individuals, but in universals and in facts whose terms are universal. "Dishonor is worse than death" is a particular case of the relation "worse than," and everything is a particular case of the logical character of "objectivity." If one means by the particularity of an object or fact simply its identity, he ought to speak of particulars of higher and lower orders, for the world is composed of identities of higher and lower orders. A fact that contains as a term some object that enters as an element of unity in another fact is of a higher order of particularity than this second fact. "Dishonor is worse than death" is not of the same order of particularity as "Judas was dishonorable"; the latter is the kind of fact from which the former is abstracted. The quality which appears as a qualifying element in the latter is itself a subject of description in the former; "dishonor" is taken now as a substantive instead of as an adjective.

In this manner, facts are overlaid, the one on the other, the less particular on the more particular, the terms of those of higher orders of abstraction being separated out from those of

lower orders of abstraction, till we reach the facts whose terms are the most abstract of all — the facts of logical structure. But any fact at any stage is particular in some sense.

The relation between these different orders is like the Aristotelian relation of matter to form. The lower orders of particulars are the matter of the higher orders; a fact which is particular in relation to one fact is general in relation to another; and the lowest members in the scale are those whose terms are finally and irreducibly particular. That is, they are facts whose terms are capable of being terms and nothing else — whose terms are individuals. These terms are like Aristotle's prime matter; they are cases of universals, they have form in the Aristotelian sense, but there is nothing in relation to which they are form. Nothing is an instance of them. They are that upon which universals rest, but which themselves rest on nothing.

Though the individual which is an instance of a universal is one with the universal in the instance, the process of abstraction — of separating the two for thought — has begun when the universal is distinguished in its instance. To abstract is to disregard an aspect of a presented object; and since all presented objects are of double aspect, both universal and individual, we can disregard the latter and refer only to the former. Not only can we abstract from the individuality of an object; we can also abstract from any of its more specific qualities or relations. Thus a *green* object is a *colored* object; when I am presented with the relation *before*, I am also presented with an *asymmetrical* relation; and I can refer to the *color* apart from the *greenness*, or to the *asymmetry* apart from the *beforeness*. The very presentation of objects — for they are only presented when they are known both as universal and individual — is the beginning of abstraction.

The abstraction which begins in distinguishing a universal in a perceived instance is completed in the realm of conception;

the instruments of conception (symbols) are also the instruments of complete abstraction. In perception the sense of the oneness of the object — as an individual in which many universals join — cannot be shut out; but through symbols any of these universals, or for that matter, the individual, can be referred to alone. We can abstract the individual, as an indeterminate x, from the universal, or the universal from the individual. The abstracted universal is not referred to as it occurs in any special one or in *all* of its instances; it is meant apart from instances, as an object with an identity of its own.

Needless to say, the symbol which carries an abstract reference is not itself abstract; it is a presented object with its own individuality and general nature. And yet the once heated controversy over "abstract ideas" turned on a confusion between the abstractness of the symbol and the abstractness of its reference. It is indeed absurd to hold, as Berkeley accuses Locke of holding, that ideas as images (as psychical events) are abstract; they are obviously as concrete as any other presented objects. But there is no absurdity in their having an abstract meaning. The problem of abstraction, so far as it touches universals, comes in the end to this: Can we refer to universals as single, self-identical objects distinct from individuals? And this is the problem of the objectivity of universals.

X

There is no reason to suppose that a universal is any the less an object than the subject it qualifies or the terms it relates. If I say that "the afternoon is warm," or that "Chicago is between New York and San Francisco," the warmth is as objective as the afternoon, and the relation *between* as genuine a constituent of the fact as Chicago, New York, or San Francisco. Unless the concept of an object is taken in a much narrower sense than we have given it, universals cannot be distinguished from individ-

ual subjects and terms on the ground that the latter are objects while the former are not. Both are in knowledge; both are given as constituents of facts, though neither is given alone.

When it is said that universals are not objective, what is meant is that they are not *in reality* the same kind of object as their terms when these are individuals; universals do not belong in the same metaphysical category as individuals. The nominalist views the world as being really a collection of individuals; what are called universals are to him ideas, mental addenda, created by an act of comparing individuals. He believes that white objects exist, but that whiteness does not exist — at least that whiteness exists only in a mind. It is a name for a number of things which are closely alike, that is, for an artificial construction, "the class of white objects."

In answer to the nominalist, it must first be observed that the question as to whether a universal exists in a mind or in reality (assuming that reality and "a mind" are different), is a metaphysical and not an epistemological question. Universals are known in some manner, whether it be as elements in an external or an internal world. They appear in the simplest units of cognition; and whether or not they are contributed by the perceiving subject, they are objects of knowledge in the broadest sense of the term. For the purposes of the positive theory of knowledge, this is enough. We need not ask if reality is a concourse of individuals, or if it is a network of universals without individuals, or if it is a fusion of both.

There are, however, many considerations that cast doubt on a metaphysics which affirms that individuals are real and that universals are unreal. First among these is the indeterminateness of our knowledge of the individual. If the x which is qualified or related by universals is the sole known reality, knowledge touches reality only at the tiniest point — only through the fleeting "this" of perception, which can be described in no

way. Secondly, it appears beyond question that when one perceives a colored object he is presented with the *color* as well as the colored object. Color is given in the field of vision, and the fact that it might be found elsewhere is no evidence that it is not here.

The nominalist will doubtless reply to this by asking how one can perceive an abstraction. The answer is that the color of an object is no more an abstraction than its individuality; the "what" of an object is no less *of* the object than its "this." The very act of perception is the beginning of abstraction; for a presented object is both broken in two for thought (that is, abstract) and knit into a unity for perception (that is, concrete).

Thirdly, to say as the nominalist does that an object belongs to a class is to notice its similarity to another object, and although the discovery of the qualities or the relations of the object comes through comparison, the result of the comparison is the perception of the universal in the object, that is, as an element of the whole. And if objects are similar, this similarity is a universal; the likeness of objects, no matter how remote, rests therefore on a universal. In order that the nominalist may maintain his point that classification is artificial, he must either believe that it is wholly a matter of caprice (and then it is not classification), or that there is some basis for it; and unless he wishes to negate his own view, he must show in the latter case that classification rests on something other than universals. This he does not show.

It can be further urged, in favor of the equal right to reality of the universal, that objects which are known as bare identities, as mere "thises" and "thats," still have the property of being *objects*. The very entrance of a thing on the stage of thought implies that it has a minimum of characters and relations — that it is identical with itself and diverse from other things.The concept of the individual itself gives rise to a universal — individu-

ality. Therefore, if universals are unreal, it ought to be added that all objects of thought, including individuals, are unreal; and the distinction between the reality of universals and individuals falls to the ground.

The nominalist will then ask, granting that one is presented with whiteness, hardness, and other sensory qualities, is one also presented with relations and operations? These seem to be much less "presentable" than qualities. But if the whole content of the perception were *a*, *b*, *c*, when *a* is perceived to be *between b* and *c*, this mere plurality would not be a perceived fact. Some sort of unity pervades and characterizes the whole; in this case, the relation "between." But perhaps it is less difficult to understand how one can perceive a specific relation than how he can perceive this general unity. Is there a perception of the unity of objects? The only answer to this is that every perception is one of the unity of objects; without a presentation of unity, which is not unlike Kant's "unity of the manifold," there is no presentation of objects. One is no more aware of objects as dis-unified than he is of a white object without its *whiteness*. The two cases are exactly parallel. They are cases of the apprehension of the universal in its union with the individual.

As for operations, such as $(2 + 2)$ — they also are kinds of group-unity. If two objects are before me, "twoness" is before me; and when I perceive four objects as a group of "two *and* two," I am presented with something which approximates to a case of the operation *plus*.

It is quite as accurate to describe the growth of knowledge by pointing out that the individual is abstracted from the universal, as it is to describe it as an abstraction of the universal from the individual. Knowledge of the individual is not prior to knowledge of the universal; to discover the individual requires an effort of thought. And if universals are "ideal," if they enter knowledge only through the working of the mind in sifting ex-

perience, the same is true of individuals. As a matter of fact, neither of these descriptions is true. The "this" and the "what" blend in the pure awareness which underlies and completes clear cognition; they emerge as distinct in the perception of objects, but the one does not thereby become less objective than the other.

To hold, with M. Bergson and Mr. Bradley, that neither qualities or relations, or the individual things qualified or related, are real — that all are "ideal" — is to gain at least the merit of consistency. Nothing short of this will do if one sets out to attack the reality of universals. But it is not necessary to prove that universals or individuals, or both, are real in order to vindicate their objectivity. If either, or both, are appearances, they are still objects; for an appearance is an object.

General propositions, laws, and principles which ignore the individual have a meaning that is as much a part of the world of fact as the meaning of propositions whose terms are bare particulars. Abstraction is possible because universals are self-identical objects which can be singled out and referred to through symbols.

XI

The analysis of facts into terms and unifying universals is the basis of a general "grammar" of symbolism. Substantive symbols (symbols of terms) cannot be taken together to form significant wholes; they must be grouped with adjectival symbols — with symbols of unity. Nor can adjectival symbols alone make up significant expressions; some of the symbols of a group must signify terms, whether these be universal or individual terms. An expression without a symbol of unity, or without terms, would violate a fundamental principle of the structure of fact, and could not possibly have a meaning among facts.

This grammar of symbolism is the source of all special grammars. Every language, if it contains no other distinctions of

function among its words, will differentiate substantives from adjectives — "adjective" being taken to include all modifying or connecting words, that is, adverbs, verbs, conjunctions, etc., as well as adjectives proper. Substantives or adjectives alone will not, in any language, enter into significant unions with one another. Thus "chair table book," "large brown on," are intrinsically without meaning, for they do not conform to the first principle of symbolic grammar. On the other hand, "book on table" or "large brown chair" are significant phrases (whether or not they stand for objects) because they are properly constructed. Language recognizes not only the distinction between terms and elements of unity, substantives and adjectives, but it recognizes also that the same object, if it is a universal, can play the part of both. A root-word assumes a different form when it enters as a term from the form it assumes as an element of unity. This is the difference between the words "nearness" and "near," "beauty" and "beautiful," "run" and "running." These distinct forms of words signify the same object, but they represent this object in different structural relations to other objects, that is, in its adjectival and its substantive functions.

The signs of addition, multiplication, subtraction, and division are the "adjectives" — the symbols of unity — in arithmetic and algebra; the signs for numbers cannot by themselves constitute significant arithmetical or algebraic expressions. They must be joined by operational symbols, for an arithmetical or algebraic fact is made up of numbers operated on in some way. And a similar "grammar" is present in symbolic logic. $(p \cdot q)$ or $(p \vee q)$ is a group in which the dot, in the one case, or the sign (v), in the other, stands for the unifying element, while the letters stand for the terms — for *propositions* united by the relation "and" or "or."

There are many special rules for the construction of significant symbolic groups, but the general rule — that symbols for

terms must be grouped with symbols of unity and that symbols for terms or symbols of unity alone cannot have meaning, if they are grouped — lies beneath all systems of symbols. Any possible symbolic group in any possible system must follow this principle, since any possible fact will be a unity of terms and qualifying or relating universals; the most general rule of symbolic syntax rests on a "syntax" of fact — on the most general way in which objects fit together to form wholes.

XII

Each symbolic system will have its own special plan of syntax; and this plan, if it assumes no very large proportions, can be set forth merely by enumerating the significant groups permitted in the system. There may be no special rules for constructing the groups, though none of the groups can violate the general principle which has just been stated. More extended and useful systems, such as language and arithmetic, are built on syntactical plans which can be embodied in rules. These rules state how the symbols can be taken together as significant wholes and how groups can be derived from one another.

Once the syntactical plan of a system is given as a set of rules, expressions of many different forms can be constructed within it, and none of these expressions will be without syntactical meaning so long as it conforms to the plan. The significance of the groups will be a function of the meanings of their elements and of their form; it is necessary only that the elementary symbols of the system should stand for existing objects; the groups may or may not represent existing objects.

Some systems, notably languages, contain an endless number of groups which fulfill the conditions of syntactical significance but stand for no objects. These systems are non-deductive (or incompletely inferential). If we follow their syntactical rules, we are as apt to be led away from fact (and truth) as to be led to

fact (and truth). In these systems, there is a discrepancy between significance and truth. But in a deductive system which is completely interpreted there is no such discrepancy. Any group that can be properly derived from the original groups by the rules of syntax will stand for an object, that is, it will be true as well as significant. The conditions of significance and of truth are the same in a completely interpreted deductive system, and the rules of syntax are the rules of deduction.[1]

But in any case, the syntactical plan of a symbolic system determines its range of significance. It determines not what objects actually *are* represented in the system, but how the symbols can be used significantly regardless of whether they stand for objects. It determines an area of concepts.

This gives us a criterion of possibility for knowledge. A "possibility for knowledge" in the widest sense need not be a reference to an object; it is any meaning that is conceivable. And any group of symbols — of words, mathematical signs, or ideas — which follows the rules of syntax in a system has a conceivable meaning; the group is a possible concept. Possibility for knowledge belongs only to concepts, and if one says that a non-existent object is a possible object of knowledge, this is only a manner of speaking. He does not mean that there is any such object to have the predicate "possible"; he means merely that a significant concept which refers to no object can be framed. "The immortality of the soul" is conceivable and therefore a possibility for knowledge; but the possibility resides not in an actual fact for which this expression stands, but in the fact that the expression has meaning. Aside from this "possibility as an object of knowledge," the test of which is conceivability, there may or may not be an absolute metaphysical possibility, such as that of Leibniz's infinity of possible worlds. But with this we are not concerned.

[1] This is an anticipation of what is more fully set forth in ch. VII.

The rules of syntax in a symbolic system therefore mark the limits of possibility for that system. (And in completely interpreted deductive systems, every possible concept is also true.) But what is conceivable in one system may have no counterpart in another. The general rule of symbolic syntax — that symbols of terms alone or symbols of unity alone cannot form significant wholes — tells us how fact in general must be conceived; it marks the most general limits of possible concepts.

XIII

The two symbolic systems whose application to fact is at once most sweeping and most familiar are the systems of the imagination (of mental images) and of language.

The habits of correct and ordered speech become the habits of correct and ordered thought, so that if a thing can be thought clearly it can be said clearly. We grow unconsciously to accept certain combinations of words as significant, and to reject others as nonsensical because they do not conform to the rules of syntax. And yet the possibilities of significant combinations in language are so wide that almost any chance grouping of words has syntactical meaning. If words are thrown together at random it is more than likely that a significant phrase will be achieved, for no phrase that follows the rules of syntax in language is without meaning.

When expressions such as "the round square," "the moon is made of green cheese," "Socrates is a triangle," etc., are rejected as meaningless, they are not strictly speaking rejected as nonsense, unless we have passed over the thin line that separates nonsense from falsity and contradiction — that divides the meaningless from the fantastic. It is true that there are no objects which correspond to these expressions, yet the expressions are not totally without meaning. The very fact that we say "a round square is a geometrical contradiction," that "Socrates is

a triangle" or "the moon is made of green cheese" are fantastic notions, shows that we grasp these expressions as concepts; otherwise we could attribute neither contradiction nor absurdity to them. A nonsensical expression, an utterly meaningless collection of symbols, could be neither false, fantastic, or contradictory.

And yet nonsense cannot be without a semblance of sense. There must be vestiges of significance in the symbols, or else they would not be symbols but mere collections of presented objects. In the pseudo-phrases, "large brown on" and "chair table book," for example, the elements are significant but the wholes are without meaning. They present the semblance of wholes but they conform to no rules of syntax. In direct contrast to this type of nonsense stand pseudo-phrases that have a seeming intelligibility as wholes since they follow syntactical rules, but that are meaningless through the presence of meaningless elements. The lines from Lewis Carroll's *Jabberwocky*, a classic of nonsense, exhibit just the proper distribution of significant elements along with meaningless elements to give the whole a structure: "O frabjous day! Callooh! Callay! He chortled in his joy." So powerful is the sense of significance as a whole that the parts tend to derive meaning through their places in the whole.

The interpretation of words — or any symbols other than images — usually takes the route of the imagination, and for this reason it has been maintained that what cannot be imagined cannot be thought, that conceivability in images is the only conceivability.

The syntax of the imagination is extremely simple. Any images that can be held together before the mind form a significant group. The distinction between terms and elements of unity is not made explicit, though one never imagines an object composed of parts which are unrelated or unqualified. There are no signs of grouping other than the mere juxtaposition of

images, and no juxtapositions of images are excluded as meaningless, though many are fantastic and absurd. Yet some images refuse to go together before the mind; there are no counterparts in the imagination for "a round square," "a curved straight line," etc.; the images corresponding to these words exclude one another's presence. It is difficult to give any reason for this exclusion; but this fact shows that the syntax of the imagination has its limits, and that there are possible concepts in other systems which cannot be translated into the imagination.

What is fantastic or absurd, or even contradictory, is not then utterly nonsensical. No properly constructed symbolic expression is void of meaning. The general grammar of symbolism marks out the most general limits of possibility for thought, following the most general lines of the structure of objects, while special plans in special systems mark out narrower limits of conceptual possibility. What is an impossibility for thought in one set of symbols may not be an impossibility in another set. It is not strange, therefore, that we can "imagine" much less than we can mean.

XIV

One class of syntactical rules must be especially noticed, that is, the rules of order. The sentence, "Brutus killed Caesar," has a different significance from, "Caesar killed Brutus," and the pseudo-group, "killed Brutus Caesar," has no significance. The spatial or temporal arrangement of the symbols enters here as a factor in the meaning; the order in the symbols mirrors an order in the fact signified. Must we then include the spatial or temporal arrangement of symbols as a feature of their logical form?

It is less difficult to illustrate than to describe what is meant by the order of a group. Facts of identical constituents may be wholly different in order, they may be of opposite orders. The fact "A precedes B" is different and opposite in order to the fact "B precedes A"; the facts "A between B and C," "B

between *A* and *C*," and "*C* between *A* and *B*" are distinguished by their orders. Most active verbs and prepositions, if they enter in groups, give us phrases or sentences representing ordered facts. "Hamlet killed Polonius" does not mean the same thing as "Polonius killed Hamlet"; "grace before meat" is something other than "meat before grace."

The simplest manner of describing order is to say that all relations have a *direction*, and that a fact whose elements are related in one direction is different (for some relations) from a fact of the same elements related by the same relation in another direction. And the usual manner of representing order in language and mathematics bears out this description. The symbols of speech and writing themselves have a direction in space and time, and it is through this that the order in the fact is signified. This manner of representing order is a convenient device of symbolism, but it does not analyze order; if the order in the fact is not spatial or temporal, it merely represents one kind of order by another.

Order is something common to groups which have a direction in space and time and to other groups which are non-spatial and non-temporal, and can only metaphorically be said to have a "direction." "Hamlet believed the ghost" is an ordered fact, but "believing" is not a temporal or spatial relation, though the order it gives to the fact is reflected by the spatial order of the symbols. What is the common structural feature that makes this reflection possible?

An asymmetrical (or ordering) relation not only unites its terms, but in uniting them, distinguishes them; it lends to each a distinctive mark, so that the whole is not of the simple form, (Rab), but of the form, $(R(Sa)(Qb))$. The whole is a group of groups, and the order is the manner in which the terms are distributed in the sub-groups. Thus $(R(Sa)\ (Qb))$ and $(R(Sb)\ (Qa))$ represent different orders of the same elements. There are as

many possible orders as there are possible distributions — in this case, only two.

Consider, for example, any fact in which the constitutive relation is a transitive verb, *e.g.*, "*A* knows *B*." The relation relates the terms, but at the same time distinguishes one as "active" and the other as "passive"; and the fact is more fully stated as, "*A*, active, knows *B*, passive." This is plainly of the form, $(R(Sa)\ (Qb))$, where R is the relation "knowing," and S and Q, respectively, the qualities "activity" and "passivity." The relation both relates and "qualifies" its terms; though this must not be construed to mean that the relation is reducible to qualities of its terms. It is always a relation; but each term, through being in the relation, assumes a special character — a character that attaches to it only when it is in the relation. Language generalizes this distinction of quality which an ordering relation imparts to its terms into the distinction of "subject" and "object." Thus in "*a* before *b*" or "*a* in *b*," *a* is the subject and *b* the object; and in apprehending the fact as ordered, we grasp this distinction of relational quality in the terms. It is apprehended as of the form, (*before* (*a*, *subject*) (*b*, *object*)); and this is different from the form, (*before* (*b*, *subject*) (*a*, *object*)).[1]

The reason why there is a "direction" from one term to another in an ordered fact is, therefore, that these terms are members of distinct groups; the terms are distinct not merely as terms, but through their places in the whole. If they take different places, though the constituents of the fact remain the same, the fact will be different. Direction in space or time is a special case of this general structural feature of groups. This description of order applies, moreover, not only to complexes of two terms,

[1] It follows from this description of order that a fact such as "*A* walks and *B* runs" has order; for the terms are united by a relation, and at the same time distinguished by predicates. The difference between this fact and "*a* before *b*" is not that the former is without order, but that the distinguishing predicates of the terms are independent of the relating relation; they are not relational qualities.

but to polyadic complexes of any number of terms, in which there is no single direction or "sense" but a number of interrelated directions. The relation 'trusteeship," for example, distinguishes the "trustor" from the "trustee," both from the thing intrusted, and all three from the "beneficiary" of the trust.[1] This tetradic relation gives an order to its terms by making each a member of a distinctive group; it is of the form, $(R\,(Sa)\,(Qb)\,(Lc)\,(Md))$. But not every term in a polyadic complex need be distinguished by an order, from the other terms: "*a* between *b* and *c*," makes no distinction of order between *b* and *c*, though it distinguishes *a* from both. There are in this complex two objects and one subject; and obviously the objects are not distinct from one another through the relation. Its form can be represented as, $(Ra(Sb)\,(Sc))$, which has only three possible variations of order, *viz.*, $(Rb(Sa)\,(Sc))$, $(Rc(Sa)\,(Sb))$, and $(Ra\,(Sb)\,(Sc))$.

When order is seen to be a matter of grouping alone, it can be represented by symbols whose spatial order is irrelevant to the meaning — whose group relations alone are significant. $(Ra(Sb)\,(Sc))$, which might mean "*a* between *b* and *c*," can be written $((Sb)Ra(Sc))$, or $((Sc)\,(Sb)Ra)$, or $((Sb)\,(Sc)Ra)$, etc.; and all of these different spatial arrangements will signify the same order of the fact. Language finds in the passive form of expression that order can be indicated by grouping alone. "Polonius was killed by Hamlet" is equivalent to "Hamlet killed Polonius"; and here the group of the agent (or subject) [2] is distin-

[1] The other distinctions of *case* in language, which are additional to the nominative and accusative (subject and object), are generalized forms of these distinctions of order in polyadic complexes.

[2] Expressions in the passive form show us, further, that the distinction between subject and object as the active and passive terms, respectively, of a relation (or as the *referent* and *relatum*) does not always correspond to the distinction between the grammatical subject and predicate of the sentence. Only in the active form is the subject of the relation — or *relational subject* — also the grammatical subject; *e.g.*, in "Hamlet killed Polonius," "Hamlet" is both the relational and grammatical subject, while in "Polonius was killed by Hamlet," "Polonius" is the relational object but the grammatical subject.

guished by the preposition "by." The expression has the form, (*Polonius killed* (*by Hamlet*)), that is, (*Ra*(*Sb*)), where *R* is the relation "killing" and *S* the preposition "by" — the mark of the agent. (*Rb*(*Sa*)) would be the alternative order. It is clear that in the passive form the spatial order of the words does not affect the significance, so long as the integrity of the groups is preserved. The sentence might read, "By Hamlet Polonius was killed," or "Polonius was by Hamlet killed," or "By Hamlet was Polonius killed," or even "Killed was Polonius by Hamlet." In any of these spatial arrangements, the meaning will be the same; and a different order can be signified only by interchanging the terms in the groups.

It is therefore only accidental, and not essential — though it is a useful accident — that the spatial or temporal order of symbols should be significant.

XV

If an asymmetrical (or ordered) fact is one in which the terms are distinguished by being members of different subordinate groups within the whole, a symmetrical fact is one in which there is no such distinction of terms through their membership in subordinate groups. Consider "*A* with *B*." The relation "with" imparts no distinctive qualities to its terms; each term is with the other and nothing more. The fact has the form (*Rab*) and could be symbolized by this group where the spatial and temporal orders of *R*, *a*, and *b* are irrelevant to the meaning. There are no subordinate groups here in which the terms could be differently distributed.

We are in the habit, however, of representing symmetrical facts as if they had an order. Since the spatial order of words and mathematical signs is usually significant, we assume that "*A* with *B*" means something other than "*B* with *A*" until the contrary is asserted. For this reason, the symmetry of such expressions as "*A* with *B*" or "$a \times b$" is commonly described by

saying that alternative orders of the symbols have the same meaning, that is, that "*A* with *B*" is equivalent to "*B* with *A*," that "$a \times b$" is equivalent to "$b \times a$." But this is necessary only because the fact, which is without order, is represented by symbols with an order. If the fact were originally represented by a group such as (*Rab*), in which the order of the symbols is irrelevant, its symmetry would be apparent. Therefore, the commutative law in algebra and logic — that $(aRb) = (bRa)$— says nothing more than that the spatial order of the symbols is irrelevant to their meaning.

The distinction between subject and object (and between agent and patient) is also carried over to expressions which stand for symmetrical facts; they are represented *as if* such a distinction were present in them. Thus "*a* is accompanied by *b*" states a symmetrical fact in asymmetrical form; but it is the same fact as "*b* is accompanied by *a*," and to recognize this is to see that no distinction of subject and object (of agent and patient) exists in the fact. The order, which is in this case a certain grouping of the symbols (rather than a spatial order), belongs only to the symbols; the fact is without order.

Mr. Bertrand Russell describes all relations as having "sense" or direction; no relation, he says, is without a *referent* and a *relatum*.[1] This is certainly true for asymmetrical relations; the *referent* and *relatum* are what language recognizes as the subject and object. But the assumption that all relations — symmetrical and asymmetrical — differentiate their terms as *referent* and *relatum* has no foundation in the data of perception. It rests only on the custom of representing all relational complexes by symbols with an order; and it obscures the true nature of symmetrical relations, for it makes them a special case of asymmetrical ones. The mere fact that *a* and *b* are related does not distin-

[1] See B. Russell, *The Principles of Mathematics* (1903), ch. 9; also, A. N. Whitehead and B. Russell, *Principia Mathematica* (1910), i, 34.

guish *b* from *a* in any way. Why we should say that *a* is the *relatum* any more than *b*, or *b* the *referent* any more than *a*, is impossible to see. A relation is a kind of unity of two terms (or more), and only some relations distinguish, as well as relate, their terms. If it is assumed that all relations have "sense," it is of course impossible to define a symmetrical relation in any other way than as one which is identical in both of its senses; and this amounts to saying that the distinction of sense (for this sort of relation) is merely symbolic. The more direct way is to represent the fact in the first place by symbols in which there is no distinction of sense, for this makes it clear that symmetrical relations are of a different genus from asymmetrical ones.

XVI

The representation of order in a fact through the spatial and temporal order of symbols is open to an ambiguity which is not present when the order is reproduced by the grouping of the symbols. There is no mistaking the meaning of "*A* is preceded by *B*" or of "Romeo was loved by Juliet." One knows that the first of these expressions means the order in which *B* is the *referent* and *A*, the *relatum;* and that the second means the order in which Juliet is the agent and Romeo is the patient. But the expressions, "*B* precedes *A*" and "Juliet loved Romeo," lack this explicitness of meaning; and if we could express the order of the fact in no other way than the latter, we should not know which of the alternative orders was meant.

The symbols permit two possible spatial arrangements — (*aRb*) and (*bRa*) — and the fact, two possible orders; but the question remains, which represents which? The meaning of these groups must therefore be further defined.

It is understood that "*B* precedes *A*" has the same meaning as "*A* is preceded by *B*;" that "Juliet loved Romeo" has the same meaning as "Romeo was loved by Juliet." Thus a unique

correlation is established, so that one spatial order of the symbols means one order of the fact, while the other means the alternative order. Without this further definition of the meaning, we should know that "Juliet loved Romeo" stands for a fact composed of *Juliet*, *loved*, and *Romeo*, taken in one of two possible orders — taken in the order opposite to that of the fact meant by "Romeo loved Juliet." But neither order of the symbols could be attached uniquely to either order of the fact.

A complete and unambiguous interpretation of the meaning of symbolic expressions requires that the spatial and temporal order of symbols be supplemented, as a means of representing order, by the more accurate and general kind of representation through grouping. (This appears in language as the syntactical rule that the relational subject of a phrase or sentence precedes the relational object, excepting where the subject and object are represented by case endings, or where the sentence is in the passive form.)

Order then is not an independent factor of logical form which refuses to be assimilated to group structure. The distribution of terms in distinct major or minor groups which make up a whole is their order. A symmetrical relation is one that groups its terms without distinguishing any of them through their membership in sub-groups; and an asymmetrical relation is one that groups its terms so that some of them are distinct not only as terms, but through their membership in sub-groups. The form, $(Rabcd)$, is a symmetrical form (the spatial order of the letters being irrelevant), and can be written indifferently, $(abcRd)$ or $(aRbcd)$, etc. Similarly, the form, $((Sa)R(Sb))$ is a symmetrical form, and can be written indifferently $((Sb)\,(Sa)R)$, $((Sa)\,(Sb)\,R)$, etc. On the other hand, $(R(Sa)\,(Qb))$ is an asymmetrical form, and permits the alternative order, $(R(Sb)\,(Qa))$, though it can be written indifferently $((Sa)R(Qb))$ or $((Sa)\,(Qb)R)$, etc., in the one order, and $((Sb)R(Qa))$ or $((Sb)\,(Qa)R)$, etc., in the

other order. Only the simpler types of symmetry and asymmetry have been illustrated, that is, those which arise through dyadic and triadic relations; but the same principles for distinctions of order hold in facts of any degree of complexity.

XVII

The distinction between the terms and elements of unity in groups raises the question as to whether a symbolic group may contain more than one symbol of unity and a fact more than one unifying element. Is every unifying element the center of a single group?

In cases where simplicity of statement is desired, each group is viewed as if it were constituted by a single relation, operation, or quality. This group may enter as a term in other groups, or may contain terms which are themselves groups, but each group — whatever its part in the whole may be — will be constituted by one and only one unifying universal. In the arithmetical expression $(2 + 3) \times ((4 + 5) - 6)$, for example, there are several different groups, each with a single element of unity. The groups are piled upon one another, and none contains more than a single operation. But it is not necessary that only a single element of unity should appear in a group.

A number of qualities, relations, or operations may act together as unifying universals in the same group. Language affords many examples of expressions of this nature. If I say, "he is poor but proud," I attribute to the subject, not a single quality, but a complex of qualities — "poor but proud." In the fact, "Boston is near to and north of New York," the terms are joined, not by a single relation, but by two relations.

The elements of unity in these facts are complex. They are themselves groups of universals united by universals; they are elements of unity which themselves contain secondary elements of unity. "Poor but proud" has the rôle of a single quality and

"near to and north of" the rôle of a single relation. The facts in which they enter are of the forms, $(Q\Phi Ra)$ and $(T\Psi Mbc)$, where Φ and Ψ represent the secondary elements of unity. Adverbs and adverbial expressions always modify elements of unity — qualities, relations, connectives — and give rise to complex elements of unity. When the forms of symbolic expressions and facts are described, the complexity of the elements of unity cannot be neglected if the description is to be complete; for this is a genuine aspect of their form. However, for purposes of simplification, a complex element of unity can be treated as a single universal; and if this assumption is made, the form $(Q\Phi Ra)$ is the same as (Na), and the form $(T\Psi Mbc)$ the same as (Kbc). But it must not be forgotten that this is an artificial simplification of the structure, and that a full description of the form would not overlook the complexity of the element of unity.[1]

XVIII

The present chapter, beginning with the distinction between universals and individuals, has examined the function of universals as elements of unity in facts, and has shown how this function is mirrored by symbols which perform a corresponding function in symbolic groups.

The individual and the universal are present in all concrete wholes of knowledge, for there is no presented object which is not at once a unique "this" and a "this" with some characters and relations. For finite knowledge, these characters and rela-

[1] The multiplicity of a group, it has been said, is determined by the number of its constituents, that is, the number of its *terms* plus one; for the element of unity (which, if it is complex, is treated as a single entity) is a constituent though not a *term* in the group. But in classifying relations as monadic, dyadic, triadic, etc., account is taken only of the number of terms they unite, and thus, according to our definition of multiplicity, a dyadic relation would constitute a triadic group, etc. This is merely a matter of terminology; the multiplicity of a group could be defined by the number of its major *terms* rather than its major constituents, so that a triadic relation would constitute a triadic group, etc.

tions do not determine the individual; and in this respect spatial and temporal relations are no exception, for the concepts of space and time are built on the perception of the numerical identity and diversity of individuals, and individuals are located in space and time only by reference to other individuals. Though it is possible to assume, with Leibniz, that an infinity of universals would determine an individual, this assumption is of no aid to finite knowledge. The distinction between the "what" and the "this" continues in its original form. The individual, however, is brought into conception through the use of a variable symbol, to which no value is assigned; but this is the only way in which the individual as such can be thought. Knowledge, in referring to the individual through a symbol of indeterminate meaning, presupposes its own indeterminateness.

The universal, on the other hand, is presented as an identity in diverse individuals — in changing and different instances. A changeless universal persists in the background of every change. Therefore the universal can be represented by symbols of fixed meaning, and knowledge in universal terms, that is, law, is freed from the variability which infects all knowledge of the individual.

Universals fill a unique rôle in groups. They are elements of unity and themselves members of the group, but not members coördinate with the others, which are terms. A relation, a quality, or an operation is not *related* to its terms; it is one with them, and at the same time distinguishable from them. The unity of a group is its "what," different universals are different kinds of group-unity, and there is no group which is not such a unity; the concept of group-unity is an indefinable but essential part of the concept of a group.

Individuals play the part only of terms; without universals they do not enter into wholes; no fact is constituted solely by individuals. But the terms of a fact, as well as its element of

unity may be universal, in which case the fact will be abstract. Abstract facts are aspects of concrete facts, that is, of facts whose terms are individual; they are separated out from the individualized data of perception. This separation begins in the perception of a universal in its instance, and is completed in the realm of conception by the use of symbols which ignore the individual and refer only to the universal apart from any or all instances. The problem of abstraction — whether universals can be thought apart from individual instances — is the problem of the objectivity of universals; for if universals are objects, they can be referred to through symbols as single and self-identical things. Nominalism, by attacking the reality of universals, does not disprove their objectivity; if universals are "appearances" or mentally constructed elements in knowledge, they are still objects of knowledge, for such appearances (or mental constructs) are objects of knowledge.

A general "grammar" of symbolism rests on the distinction between terms and elements of unity. Symbols of terms alone (substantives), or symbols of unity alone (adjectives), cannot constitute significant expressions; and this fact is recognized in all symbolic systems — in language, in mathematics, in logic. Possibility for knowledge is conceivability in a symbolic system; and any possible object of knowledge must be composed both of terms, which may be individual or universal, and of unifying universals. Special plans of syntax mark the limits of conceivability, of possibility, for special systems, and what is possible in one system may have no counterpart in another. Thus no expression that follows the rules of syntax in a system is nonsense, though its meaning may be fantastic or contradictory; and a possible concept may or may not refer to an actual object.

Rules of order are a special class of rules of syntax. The spatial and temporal order of the symbols is significant in most systems; it reflects an order in the fact. But order can be described

and represented through group structure alone, since an ordering relation (or operation) is one that both unites and distinguishes its terms, this distinction arising through the membership of the terms in different sub-groups within the whole. Order must be represented through group structure if all ambiguity of interpretation is to be eliminated.

Finally, the elements of unity in groups may themselves be complex; they may be groups composed of universals united by a secondary element of unity.

The study of grouping, of the manner in which elements determine wholes, both in the fact and in the symbols, is necessary to the understanding of any expression. It is here that nuances of meaning are hidden. Sentences made up of many qualifying adverbs, adjectives, phrases, and clauses are susceptible of subtle shades of interpretation, or misinterpretation, according as they are taken in one or another of several possible groupings. The group structure, the form, is *presented* in the symbols and is immediately apprehended in its entirety, as is a musical phrase or a visual design. No description of the intellectual processes is better than Plato's — "the contemplation of forms"; for forms do not arise without universals, and they are themselves universals, present in thought, through which the mind fastens on its objects.

CHAPTER IV

DESCRIPTION AND ANALYSIS

I

THERE are two ways of referring to objects — by single words or symbols and by descriptions. A description is a phrase such as "the bard of Avon," "an enemy of the people," "some member of Parliament," "the divine right of kings." The description may be either ambiguously or unambiguously interpreted, and it may refer either to an individual or a universal. In any case, it signifies an object through some of the predicates (relations or qualities) of the object, and it is a different form of reference from a single symbol, which might mean the same object.

Mr. Bertrand Russell, who has thoroughly analyzed the nature of descriptions, believes that a description has meaning only in use; that it is an incomplete symbol which, apart from some proposition in which it occurs, is meaningless; [1] while on the contrary he believes that single words and proper names are complete symbols; they have significance in their own right, either in or out of a context. Mr. Russell's theory of descriptions rests on a conception of meaning wholly different from the one developed here. It can be shown that descriptions have syntactical meaning, whether or not there is an object for which

[1] In the *Principia Mathematica* of Messrs. Whitehead and Russell, however, the term "descriptive phrase" is used to cover only expressions preceded by "the"; it is merely these expressions which are said to be incomplete symbols, *i. e.*, to have meaning only in use. For the purposes of unifying and simplifying the present discussion, we shall speak of all phrases preceded by the particles "the," "a," "any," "some," "all," "each," and "every," as descriptive phrases. But it must be remembered that what Mr. Russell says of descriptive phrases, in the *Principia Mathematica* at any rate, applies only to expressions preceded by "the."

they stand, and that their significance does not depend on their context.

In *The Principles of Mathematics*, Mr. Russell deals with descriptions under the head of denoting phrases. "The notion of denoting," he says, "like most notions of logic, has been obscured hitherto by an undue admixture of psychology. There is a sense in which *we* denote, when we point or describe, or employ words as symbols for concepts; this, however, is not the sense that I wish to discuss. But the fact that description is possible — that we are able, by the employment of concepts, to designate a thing which is not a concept — is due to a logical relation between some concepts and some terms, in virtue of which such concepts inherently and logically *denote* such terms. It is this sense of denoting which is here in question. . . . A concept *denotes* when, if it occurs in a proposition, the proposition is not about the concept, but about a term connected in a certain peculiar way with the concept. If I say 'I met a man,' the proposition is not about *a man:* this is a concept which does not walk the streets but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank account or a public-house and a drunken wife." [1]

Mr. Russell does not here explicitly take the position that the denoting or descriptive phrase has no meaning apart from a proposition in which it occurs; this is a later addition to his thought. But the problem he puts is the one that must first be faced in examining the nature of descriptions.

The object meant by a description is a term and not a concept or mediating entity of any sort; it is a term referred to through universals which cluster about it. The concept means

[1] B. Russell, *The Principles of Mathematics*, 1903, ch. 5. In a later statement on denoting, Mr. Russell abandons the idea of a relation by which concepts inherently and logically denote terms and states his theory of descriptions without this presupposition. See, *Mind*, N. S., 1905, xiv, 479 ff.

the term indirectly through predicates or relations of the term, and it does not mean simply these predicates or relations. "The friend of Caesar" does not mean a concept, nor does it mean "friendship for Caesar"; "a man" does not mean "manhood" or "manliness"; "an enemy of the people" does not mean "enmity for the people." Each of these phrases means *that to which* the predicates attach; the term is signified through universals which modify it.

II

Now universals always appear in instances; they either modify individual terms directly, or modify universal terms which are themselves attached to individuals; and the "relation" in virtue of which some concepts inherently and logically denote some terms is that they refer to universals which are in unity with these terms, so that the terms are "an instance of" these universals. The unity of fact is such that any predicates which attach, let us say, to Shakespeare are genuine aspects of this individual, whether or not they completely exhaust his individuality. "Bard of Avon" is therefore capable of denoting the term with which it has this unity; "man" is capable of denoting any individual of which "manhood" is a predicate; every reference to a universal or complex of universals is capable, inherently and logically, of becoming a reference to terms.

A descriptive phrase rests on the knowledge of a fact in which the object described appears as a term. An object known to have such and such predicates or relations can be described as "the" object or "an" object of these predicates and relations. The unity of terms with their predicates makes this possible. But the description is not tantamount to a statement of the fact in which the term appears: to say that "Shakespeare wrote *Romeo and Juliet*" is not to describe Shakespeare as "the author of *Romeo and Juliet*." The description is based on a knowledge of the fact,

and includes a reference to the fact, but it is something other than a statement of the fact. "Shakespeare wrote *Romeo and Juliet*" means that there is a certain relation between certain terms. "The author of *Romeo and Juliet*" means one of these terms — Shakespeare; though in meaning this person, it also means in some sense that this person wrote *Romeo and Juliet*.

There seems to be a reason, therefore, for distinguishing between denoting or descriptive phrases and propositions — propositions being references to complex wholes of terms, predicates, and relations, and denoting phrases being references to terms alone, though not without some accompanying subordinate reference to complex wholes in which the terms occur. In the traditional terminology of logic, a description *denotes* a term and *connotes* a proposition about this term; "the author of *Romeo and Juliet*" denotes Shakespeare and connotes Shakespeare's writing of *Romeo and Juliet*. But this separation of connotation and denotation throws no light on the problem; it merely distinguishes two sorts of meaning which, once they are admitted as distinct, cannot be brought together under any single concept of meaning.

Mr. Russell undoubtedly has in mind some such distinction, though he does not speak of "connoting," when he says: "A concept *denotes* when, if it occurs in a proposition, the proposition is not about the concept, but about a term connected in a certain peculiar way with the concept." Without doubt the proposition, "I met a man," is not about the concept "a man" or about "manhood." It refers to a person, an individual. Nor is the proposition, "I have seen the portrait of the author of *Romeo and Juliet*," a reference to some mediating entity, to some concept, or merely to the authorship of this play; it is about Shakespeare. And yet each of these propositions is about more than a bare term. It is about the term as it is qualified in a certain way, that is, as "an" instance or "the" instance of a

predicate. "I met a man" means more than "I met x"; it means "I met x and x is human." "I have seen the portrait of the author of *Romeo and Juliet*" means more than "I have seen Shakespeare's portrait"; it means this, and "that Shakespeare wrote *Romeo and Juliet*." If the proposition is not about a predicate taken in abstraction from a term, neither is it about a term taken in abstraction from a predicate. It means the predicate as it is particularized in a term.

A descriptive phrase cannot, therefore, be distinguished from a proposition on the ground that a proposition means a complex whole while a descriptive phrase means only a single term. A descriptive phrase means, as any complex of symbols means, through the meanings of its elements, and it means a complex whole. It sets up a complex intention, which is a function of the simple intentions of its elements; and although it does not *state* the fact on which the description is based, it refers to this fact. It is misleading to say that a descriptive phrase denotes a term, and that this exhausts its meaning. The phrase *means* (and where connoting is not distinguished from denoting, it *denotes*) the predicates through which the term is described no less than the term; the term could not be signified through predicates unless the predicates were also signified.

III

A descriptive phrase differs from the proposition on which it is based through its form. It is no less a reference to a complex object, to universals as they enter into unity with terms, than the proposition; but its form, unlike that of the proposition, is one in which a term — *the* term described — takes a central position, so that this term receives an emphasis it would not have in a different structure of the same elements. The term is viewed (is placed in a group) as a subject qualified by certain predicates — predicates which may be simple qualities such as

"humanity" or a complex set of relations to other terms, such as "author of the greatest tragedies in English literature." The group revolves about the term. At the same time, this term is symbolized by a variable, which in language is presupposed rather than explicitly presented in the symbolism. Thus a description is a variable reference to a particular instance of a universal or of a complex of universals and terms.

A particular instance of a universal is a "this of some sort" or a "this in certain relations"; it is an object of the form (xR), where R is the universal (or the complex predicate) and x is a term. The particles which precede descriptions inform us that the expression is to be construed as meaning an object of this general structure. "The friend of Caesar" is to be interpreted as standing for x as qualified by friendship-for-Caesar; "the author of *Waverley*" to mean y as qualified by authorship-of-*Waverley;* "a man" to mean z as qualified by humanity.[1] But the particles also tell us more than this; they tell us that a variable is present in the symbolism, and that an interpretation for this variable is to be chosen in a certain way.

A descriptive phrase refers then to a fact or complex object, one of the constituents of which (in language) is not explicitly mentioned in the phrase but is understood through the use of the particle, and this constituent is the term — the x, y, or z — described. The very omission of the term focuses the attention on it. Such a phrase is, in one sense at least, an incomplete sym-

[1] The central position of the term described can be indicated by writing it, with different typographical stress, both outside and inside the descriptive expression. Thus "the friend of Caesar," which means an x as modified by the whole complex "friend of Caesar," could be written "the x ((x friend) of Caesar)." This shows that the entire expression, as well as the single predicate "friend," modifies the term described. In our general schematism, this description is of the form $(x\,((xM)Sa))$; that is, the predicate M attaches immediately to x, but at the same time x is modified by the whole complex predicate of which M is merely one element. In short, the entire structure pivots about x. If the complex predicate $((xM)Sa)$ is represented simply by R, the form becomes (xR).

bol, but not in Mr. Russell's sense. It is incomplete not because it is meaningless outside a context, but because there is no symbolic element in it which corresponds to the term described. This element must be supplied in interpreting the expression; and it is always a variable.

The discussion of the form of descriptive phrases leads first to an examination of the nature of a variable, and secondly to a reëxamination of the concept of the analysis of objects; for a descriptive phrase is analytical (and variable) in meaning, and in this respect it is distinguished from a proper name or a single symbol, which is unanalytical in meaning.

IV

Nothing more is known of the term described than that it is a term modified by certain predicates. The x or y which the particle preceding the description permits us to supply is a symbol of whose meaning we are ignorant. It may stand for anything at all.

"In mathematical logic," says the *Principia Mathematica*- "any symbol whose meaning is not determinate is called a *variable*, and the various determinations of which its meaning is susceptible are called *values* of the variable." [1] It is apparent that a variable cannot be without meaning, for if this were the case it would not be a symbol; there is no such thing as a meaningless symbol. And yet, if the meaning of a symbol is undetermined, does this not amount to being without a meaning? Apparently there is an important shade of difference between that which is meaningless and that which is indeterminate in meaning.

The difference is this: if I know that a mark or a sound — or any other presented object — is to be interpreted as a symbol,

[1] Whitehead and Russell, *Principia Mathematica*, i, 4.

but do not know what interpretation is to be put on it, this mark or sound is a variable. The phenomenon is analogous to the significance present in the words of a foreign language incompletely understood. A sufficient number of the words are grasped to enable us to know vaguely what the ones that are not grasped mean; the latter are treated as symbols, and yet they do not have a determinate meaning. Our attitude toward x, as it appears in the expression, "x is human," is exactly this: if the whole is to be significant, x must have a meaning. We are logically compelled to think of x as a symbol, just as we are logically compelled to think of the words which are not understood in the foreign phrase as symbols; but though x is a symbol its meaning is undetermined.

A variable is therefore indeterminate in meaning because it is uninterpreted. It functions as a symbol because we know — not through knowing its meaning, but on other grounds — that it is symbolic; and yet, being symbolic, it means nothing in particular.

Variable symbols do not arise in knowledge merely through the accident of ignorance; the analogy of the uninterpreted words in the foreign language is not perfect. Variables are introduced for a purpose. We replace the constant "Socrates" in "Socrates is human implies Socrates is mortal" by a variable A, and thus achieve generality of reference — "A is human implies A is mortal." Variables are instruments of thought employed for a definite end. But it is essential to this end that no specific reference, no special intention, be attached to the variable; it must continue uninterpreted; it must be used with an intended ignorance of its meaning.

The circumstance which permits the introduction of variables is that symbolic groups have a structure and a meaning as a whole. A variable presupposes a context — a context of interpreted symbols or a general schematism of symbolic grouping;

and thus it is the truly incomplete symbol. In this context, the variable, by virtue of being a part of a whole which is taken to be symbolic, is itself taken to be symbolic. The meaning of the whole tends to lend significance to the parts, which have no specific meanings, that is, which arouse no intentions directed toward objects; and this derived (variable) significance is a new and peculiar kind of meaning, different from syntactical meaning or from the direct reference to objects through simple symbols. Out of its context a variable is meaningless; it is not a symbol and not a variable. In its context it has meaning, for if any group of marks or sounds is construed as being significant as a whole, the elements must also be construed as being significant. This is a structural necessity.

Take any significant group of symbols — a phrase, a mathematical or logical expression — the elements of which have a determinate meaning, and replace these one after another by elements which in themselves mean nothing. The substituted (and intrinsically meaningless) elements will become variables, and the group as a whole will continue to be significant so long as it is construed as a group. If in the sentence, "San Francisco is in California, and California is in the United States, implies San Francisco is in the United States," I insert the otherwise meaningless marks, x, y, and z, for the proper names, the phrase becomes, "x is in y and y is in z implies x is in z"; and x, y, and z are now variables. The phrase does not lose its meaning as a whole, but becomes less determinate; it becomes more general. But since the words "in," "and," and "implies" still have a fixed meaning, the statement is determinate for these, and indeterminate only for x, y, and z. When all of the constituents of a symbolic expression are replaced by variables, the limit of indeterminateness is reached. The only constant element of meaning that remains is the form, but so long as this form is not destroyed the expression continues to be significant as a whole. Its mean-

ing is highly general, for it is restricted to no special object, but only to objects which exhibit this form.

Only the most abstract kinds of thought make use of symbolic groups in which all the constituents are variable and the form alone is constant.[1] But if a general schematism of grouping — a plan of syntax — is given, so that any group which falls within the plan can be treated as a significant whole, this condition will determine a sufficient framework for the use of variables as instruments of meaning. Thus $((Rab)S(Qdefg))$ is a significant complex in which all the constituents are variables, provided the parentheses are taken as signs of grouping, the small letters as symbols of terms, and the capitals as symbols of unity. With these assumptions, this expression is of determinate logical form; but without the minimal context of a symbolic structure, the letters would not be variable symbols, but mere letters.

The form of the expression in which a variable occurs therefore places certain general limitations on the interpretation of the variable. Unless the structure is to be destroyed and the significance of the whole to disappear, some of the constituents must be interpreted as terms and others as elements of unity. There must be some indications in the symbolism as to which of the variables are to be thus construed. In "x is human implies x is mortal," x must be taken as a term, or substantive; in "Socrates is R implies Socrates is S," R and S must be construed as elements of unity, or adjectives. Universals in their adjectival function are not possible values of variable *terms*, and substantives (either individual or universal) are not possible values of variable symbols of unity.[2] Without this restriction, any interpretation would be formless and meaningless.

[1] See ch. VII for an example of such a system and for a discussion of "functional" variability, which is different from the "interpretational" variability here treated.

[2] This is the idea which lies behind Messrs. Whitehead and Russell's theory of types. See *Principia Mathematica*, vol. i, ch. 2 of the Introduction.

Aside from this general limitation imposed by the form, the variable can stand for any object, or it can be given a meaning which is a reference to no existent object; it can be defined as equivalent to a symbolic group which stands for nothing. The x in "I saw x yesterday" may mean "the prince of fairies"; the y in "y is human implies y is mortal" may mean "the devil" or "the god Pan." [1]

However the variable is interpreted, it must be interpreted univocally; otherwise the principle of identity would be violated. The same symbol can have one and only one meaning; and if an undetermined x is construed in one way in one context, and in another way in another context, it is not the same symbol. In "x is human implies x is mortal," the two x's, if they are given values, must be given one value. In the *Principia Mathematica*, this is stated as the special principle of "the identity of the variable"; but this principle is not different from the law of identity in its most general form as a principle of symbolism. If x is the same symbol as x, whether its meaning be constant or variable, the two must have a single meaning. For this reason — that a symbol can stand only for a single (self-identical) object — an ambiguous expression such as "a man," in "I met a man," *can* mean one and only one individual if it is interpreted, but, being uninterpreted, it does as a matter of fact mean no individual. This raises the question of the ambiguity of the variable.

[1] The borrowed significance of the variable symbols in knowledge is much like that of the nonsense words in Lewis Carroll's "O frabjous day! Callooh! Callay! He chortled in his joy." And if one gives way to the inevitable drift of significance which is present in these nonsense verses, he finds himself treating "chortled" and "frabjous" as variables — as words which must have a meaning; he is left free to interpret them as he chooses, provided he remains within the structure of the groups. "Frabjous" is evidently an adjective, and "chortled" a verb; this is fixed by the logical form. But the charm of the verse is that nothing more is fixed; within these limits the words might mean anything whatsoever.

V

Psychological ambiguity or equivocation is the use of a word in several senses at once, that is, with a number of different intentions; and it is distinguished from the univocal use of words, which is their use in one sense only, that is, with a single intention. Clearly, logical ambiguity is not equivocation. A variable does not actually set up an intention; it actually has no reference until it is interpreted. It is ambiguous in the sense that it is *susceptible* of many interpretations, it can be taken to stand for many different objects or its value can be defined in many different ways, though none of these values is necessarily its interpretation. An equivocal symbol, on the other hand, is susceptible of no interpretations but the ones that already go with it. Each of its interpretations is a constant reference and some one of them *must be* taken, in any case, as the meaning. An equivocal symbol is never variable. Consider this Shakespearean pun: "Adam was the first that ever bore arms. The Scripture says, he digged: could he dig without arms?" [1] "Arms" is not a variable, it is not equivalent to an x; if it had no determinate meanings, there would be no pun.

Thus a variable is ambiguous — is open to many interpretations — for the same reason that it is a variable, that is, because it is actually uninterpreted. Its values are "possible interpretations"; they are objects which might be but are not meant, or intentions which might be but are not aroused by it. The psychological effect of a variable, *e.g.*, in "I saw x yesterday," is to cause one to run over in his mind a number of meanings which could be attached to x and still allow the sentence to be significant as a whole; but the mind stops at none of these, for x does not mean some one of them. These meanings are possible interpretations of x; x has no actual interpretation.

[1] *Hamlet*, act V, scene 1.

The variable may be treated, however, as if it had only one possible value, and in this case it will not be a means of generalization, as it is when it has many possible values. It will be a variable with a constant (though undetermined) value, an unambiguous variable. And this is the difference between the meaning of "the" and of "a," "an," "any," and "some."

These particles, which precede descriptive phrases, are *signs of interpretation*. They do not signify constituents of the fact (if the expression stands for a fact), but they mean that a variable is present and that this variable is to be interpreted in a certain way. Each of the particles indicates a different manner of interpretation.[1]

The particle "the" signifies that there is only one possible value of the variable. The meaning of the description, "the center of gravity of the solar system," is x as modified by this complex predicate, where it is understood that there is one and only one possible x. But how can a variable have no more than one possible value and remain a variable? Does it not then become a constant? Clearly it does not, for the x is a variable so long as that single value is undetermined. If one says that "the center of gravity of the solar system is a constant," he does not mean that this object is determinately (invariably) referred to; he means that there could be no more than one object which this description signifies. (The particle "the" is sometimes used as equivalent to "a." We say that "Brutus was the (a) friend of Caesar, and Anthony was the (a) friend of Caesar," where we do not mean that either was the only friend. This is a loose use of "the", and it is evident that in the phrase, "the center of gravity of the solar system," it is not to be construed in the sense of "a.")

The particles "a," "an," "any," and "some," on the other

[1] "Every" and "each" must be treated separately since they involve the idea of "all," that is, of reference to a class.

hand, precede descriptions in which the variable is to be ambiguously construed, that is, as susceptible of many possible meanings; and this is the sole difference between a description such as "the bard of Avon" and "a bard of the sixteenth century." Both expressions are of the general form (xR), though x is not explicitly given in the symbolism; both refer through an undetermined symbol to a term as it is modified by a complex predicate. But the one specifies in advance that the variable has only one possible value, while the other specifies that it has many possible values. The one is an indeterminate but unambiguous reference, the other, an ambiguous (and therefore indeterminate) reference to a term.

A description introduced by the particle "a" or "an" can be construed as if it meant either "any" or "some"; but if the particle "any" or "some" is present, a further condition of interpretation is added to the general condition of ambiguity which is signified by "a" and "an." If I say, "a woman has the same right to education as a man," I mean *any* woman and *any* man; but if I say, "I saw a woman and a man on the street yesterday," I do not mean *any* woman and *any* man; I mean *some* woman and *some* man. And either of these unexpressed meanings can be read into a description prefaced by "a" or "an."

The particle "any" indicates that a value chosen as the interpretation of the variable must be chosen *at random* from among the possible values; there must be no principle of selection. Thus "any man" means "a man" where there is no reason to suppose that one of many possible interpretations is preferred to another. "Some man," on the other hand, means "a man" where the interpretation is not chosen at random, but by some principle of selection not stated. "Some" is the antithesis of "any," though both are ambiguous manners of reference, that is, the phrases in which they occur may have many differ-

ent values. But "some" differs from "any" in implying a specific condition according to which the value is to be chosen. "Any" implies that there is no such condition.

If I say, "Some man is your friend," I am speaking ambiguously. There are many possible values of "some man"; and yet I do not mean that "any man is your friend," for it is assumed that the choice is limited in some way. What I mean is: "Some man is your friend, and not any man, because you care only for good men," or "because your tastes in friends are not catholic," etc. There is a presupposed but unexpressed condition of choice. And if I say, "Not some man, but any man, is your friend," the implication is exactly the opposite: that there is no condition of selection.[1]

Whether a descriptive phrase be ambiguously or unambiguously interpreted, whether it be preceded by "the," "a," "any," or "some," it must, if it is assigned a meaning, be given one and only one meaning; it must be univocally interpreted, otherwise the principle of identity would be violated. And this is the paradox of the variable: that, being capable of many possible determinations (excepting when it is preceded by "the"), being an instrument of generality of reference, it must, nevertheless, be given no more than a single meaning, if it is given a determinate meaning.

VI

A term described need not be an individual. "The divine right of kings," "the royal color," are variable (and unambiguous) references to universals; while "a talent for music," "any love of beauty," "some hope of immortality," are variable (and ambiguous) references to universals.

Universals no less than individuals are instances of universals;

[1] "Some" carries no reference to *existence* as a necessary part of its meaning; it is not to be distinguished from "any" in this respect. "Some angel's soul" does not mean that "there exists an angel's soul."

and for this reason, they can be described through their predicates and relations. A universal need not be particularized in individuals alone; it can be particularized in other universals, which are in their turn particularized in individuals. The universal described is referred to in abstraction from any or all of its individual instances; it becomes the value of x in the description. This x, being a term however, will mean the universal in its substantive rather than in its predicative or relating form. "The divine right of kings" means (unambiguously) a right of a peculiar sort; "any interest of mankind" means (ambiguously) an interest of a special sort; but neither means an individual.

A described universal is thus signified through a variable expression, as is a described individual; but a universal is no more completely determined for thought through description than is an individual. A description of "friendship" to a man who had not had a friend, or of "color" to a man who had not seen color, would contain a residuum of uninterpreted meaning which could not be eliminated by description alone. And yet universals differ from individuals in that they can be known as spreading out through many perceptual wholes; they can be identified and re-identified as persisting elements in the process of change; they can be distinguished and redistinguished from other universals. But once an individual has passed from perception, it cannot be again identified as the same individual. This is why universals can be determinately referred to through single words, if not through descriptions. Every universal, since it is self-identical,—itself and nothing else,—has its distinctive perceptual flavor; description points the way, but does not take the place of the presentation of these universal objects.

VII

There is an apparent line of separation between descriptions and proper names or single symbols, and in Mr. Russell's theory

this separation becomes extreme. Mr. Russell believes that proper names (or single symbols) have meaning both in and out of a context, while descriptions have meaning only in a context.

"By an 'incomplete symbol,'" says the *Principia Mathematica*, "we mean a symbol which is not supposed to have any meaning in isolation, but is only defined in certain contexts. . . . This distinguishes such symbols from what (in a generalized sense) we may call *proper names*. 'Socrates,' for example, stands for a certain man, and therefore has meaning by itself, without the need of any context. If we supply a context, as in 'Socrates is mortal,' these words express a fact of which Socrates himself is a constituent. But in other cases this simple analysis fails." [1] These are the cases in which a phrase such as "the author of *Waverley*" is employed. Such expressions in themselves are meaningless; they have meaning only in use.

This theory is contrary to ordinary interpretations of language. "The author of *Waverley*" is capable of standing on its own feet as a significant symbol; one feels no need of a context to give it a meaning. The phrase is used in speech as if it were equivalent to the proper name "Scott," and like the proper name, it is construed as significant both in itself and in a context. Mr. Russell's view is not supported by common sense or usage; it is proved by a number of ingenious arguments which raise fundamental questions as to the nature of meaning. An examination of these arguments discloses a wholly different way of distinguishing between descriptions and proper names (in a generalized sense). The former are analyzed references or meanings; the latter, unanalyzed references. Neither depend on their contexts for significance. At the same time, we discover that a reference to an individual through a proper name is no more determinate than a reference to an individual through a description.

[1] Whitehead and Russell, *Principia Mathematica* (1910), i, 69.

One of Mr. Russell's most plausible arguments for the view that descriptions have meaning only in use is the following: "Take, for example, the following proposition: 'Scott is the author of *Waverley*.' This proposition expresses an identity; thus if 'the author of *Waverley*' could be taken as a proper name, and supposed to stand for some object *c*, the proposition would be 'Scott is *c*.' But if *c* is anyone except Scott, the proposition is false; while if *c* is Scott, the proposition is 'Scott is Scott,' which is trivial, and plainly different from 'Scott is the author of *Waverley*.'" [1] The conclusion is that "'the author of *Waverley*' cannot mean the same as 'Scott,' or 'Scott is the author of *Waverley*' would mean the same as 'Scott is Scott,' which it plainly does not; nor can 'the author of *Waverley*' mean anything other than 'Scott,' or 'Scott is the author of *Waverley*' would be false. Hence 'the author of *Waverley*' means nothing." Such expressions have a meaning in use, but not in isolation. In use, *e.g.*, in the proposition, "Scott is the author of Waverley, "the phrase becomes significant when the proposition is understood to mean "'*x* wrote *Waverley*' is always equivalent to '*x* is Scott.'"

There is no doubt that propositions in which descriptive phrases occur can be translated into propositions in which these phrases do not occur. A descriptive phrase can always be eliminated. But this is a symbolic device, and not a proof that the descriptive phrase in itself has no meaning. Usage holds that "Scott" means the same as "the author of *Waverley*," that both are significant in isolation from a context, and that "Scott is the author of *Waverley*" is not, on this account, either a trivial or false proposition. Common usage employs a descriptive phrase interchangeably with a proper name which it takes to stand for the same object, and common usage can be defended.

The judgment, "Scott is the author of *Waverley*," is analytic

[1] *Op. cit.*, p. 70.

in form. It is of the form, $x = (yR)$, which expresses an identity of meaning between two symbols, or two concepts. But the symbol on one side of the identity (if it refers to an object) refers to the object as a single and unanalyzed thing, while the symbol on the other side refers to it as a group of elements — as a complex whole which can be broken up into parts. By substituting x for (yR), which must be possible if these are identical in meaning, the identity $x = x$ can be derived, and this is trivial: it is the law of identity, which is presupposed in the use of x as a symbol. But this does not necessitate the conclusion that either x means something other than (yR) or (yR) is meaningless. The conclusion to which it does lead is this: that x (if it means an object) means the same object as (yR), but means it in a different way; that the same object can be meant in two different ways, that is, it can be referred to analytically or unanalytically; and that the exhibition of an analysis — or of different analyses of the same object — is never trivial. If "Scott is the author of *Waverley*" is either trivial or false, on the usual interpretation of these symbols, the equation $2 + 2 = 4$ is also trivial or false, for it is an analytic judgment of the same general form. It asserts that what is meant by the complex $2 + 2$ is the same as what is meant by the symbol 4; it permits the substitution of 4 for the complex $2 + 2$, and yields the identity $4 = 4$.[1]

VIII

The judgment is accused of being trivial because it seems not to be synthetic; it seems to add nothing to its subject through its predicate. But a judgment which is analytic *in form* can be synthetic *in effect*, when one side of the identity expresses an analysis of what is represented on the other side without analysis (or through a different analysis).

[1] This assumes that the equations of arithmetic represent identities, which is, we believe, the correct view.

Single words or symbols can mean a great deal more than we are explicitly aware that they mean, and this is because we can refer to an object as a whole without referring to the parts or aspects of the object. To describe an object, which is originally signified by a single word, is to add to our knowledge a knowledge of the fact that the object is composed of elements. To signify the same object in these two different ways is expressly to state that the object has certain lines of structure, that a certain analysis of it is possible. Now one can be aware of an object without expressly representing to himself its lines of structure, its possible analyses. Therefore, such a judgment of identity, though it takes the analytic form, $x = (yR)$, is synthetic in effect. It asserts more than $x = x$. Statements of identity that present an analysis of what is meant without analysis by a single symbol are the only means by which indefinite concepts can be made definite, the only way in which meanings can be defined.

But is a description an analysis? If analysis is breaking up an object into its parts, is not the discovery that an object is an instance of a universal, or that a certain description fits it, something other than analysis?

I can analyze an object into parts, a, b, and c, which are related in a certain way. I can discover that my ink-well is composed of a small glass jar with a glass cover, and this seems to be genuine analysis. But if I find that the ink-well is black and that it is on the table, I am discovering relations and qualities which attach to it, and these do not seem to be parts in the same sense. And if I now describe it as "a black object on the table," I do not appear to be analyzing the ink-well.

This is a superficial distinction. Whether its blackness and its position on the table are "parts" of the ink-well depends on the point of view. These may be, on one definition, external to the object and, on another definition, internal to it. If by "the ink-well" I intend the bare x, the individual term — what Locke

would have called its "substance" — then the fact that it has a glass container and a small glass cover is as external to it as the fact that it is black and on the table. But I can mean by "the ink-well" either the individual, the x alone, to which certain predicates attach, or I can mean the individual taken with more or fewer of its predicates and relations. And this is possible because the individual, though it may not be completely determined by its predicates and relations, is continuous with them. They enter into the individual. Any predicates or relations of an object are "parts" of the object in a logical sense, for they *participate in* the object; and when the object is described, it is analyzed as a complex whole in which these qualities and relations are elements. The bare individual, the x, of which the predicates and relations hold, becomes one element along with others.

Description is then analysis, but it may be analysis which leads to an enlargement of concepts — of the meanings of single words or ideas, or of groups — through the discovery of new characters and relations that can be included in the definition. If I make the judgment, "Scott is the author of *Waverley*," in ignorance of the fact that Scott wrote *Waverley*, what this judgment tells me is that I can enlarge my concept of Scott to include the authorship of *Waverley*.[1] I *must* so enlarge my concept if the judgment is to be true; for if I do not now mean the same thing by "Scott" as by "the author of *Waverley*," I am certainly in error.

The meaning of a single symbol can be indefinitely expanded to take in more and more of the predicates and relations of an object, so that judgments of the form, $x = (yR)$, continually widen knowledge by giving new definitions, new analyses, of

[1] This explains the surprise of George IV (mentioned by Mr. Russell) when he learned through a mere statement of identity that Scott was the author of *Waverley*.

concepts which have hitherto been used with a restricted or unanalyzed meaning.

IX

The most restricted possible concept of an individual is that of an x—a mere "this"—devoid of all characters and relations, a concept which is framed only by abstraction from the concrete, qualified, and related data of perception. Scott, though a single word is used to refer to this person, is a complex object. He has many aspects beyond his bare individuality, he is an individual of this sort or that sort, and this has been shown to be true of all individuals. Every individual is a vortex of characters and relations. When one uses a proper name for an individual, he cannot in the beginning mean an x, shorn of all relations and characters, for no such thing comes into experience to be named. "Scott," at the very least, originally means an (xR), a complex of predicates individualized in a term. Thus, from the outset, a proper name is equivalent in meaning to a minimal description of the object to which it is applied. Though the bare individual is an element in the whole, it is the whole which is named, and not the individual alone. To break up this whole and take the meaning of the proper name to be the x to which the characters and relations attach, and only this, is to reduce the significance of the name to that of an x. "Scott," if it means merely the individual, a "this" which is distinct from all other individuals and absolutely unique, is a variable; and the fact that the name is treated as if it had one, and only one interpretation, does not eliminate this indeterminateness, for it does not tell us what this interpretation is.

No view of knowledge is more inadequate than an extreme nominalism, which holds that individuals can be picked out, apart from their characters and relations, and referred to through determinate concepts, or given distinctive names. And

yet the opposite extreme is equally inadequate, *viz.*, that the individual, or any other object, is nothing but its characters and relations. The attempts to reduce terms (whether individual or universal) to their predicates and relations, and the inverse — to reduce predicates and relations to their terms — are destined to failure because they destroy the logical frame-work of fact. The theory of "internal relations" cannot show that terms can be dispensed with in describing the world, without abolishing all distinctions and all description. If all relations and predicates completely absorb their terms, everything becomes one and indistinguishable. But it is possible to interpret the "internality of relations" in another way. The wholeness of fact is such that the element of unity — the predicate or relation — *enters into* the terms as well as into the fact. This unity of the terms with the predicate or relation leaves *both* distinct for knowledge; it is not inconsistent with the externality or distinctness of terms and relations which is necessary to logical structure. But at the same time, the unity of predicates with their terms makes it possible to include in the definition or description of a term a reference to more or fewer of its predicates and relations.

Where, it will be asked, does this extension of concepts through the description of objects by new characters and relations end? It would seem that a meaning must eventually become coextensive with all the predicates of the object meant, that "Scott" or any other name must finally mean an object taken with an indefinite number of predicates. No characters or relations of the object would then be external to the meaning. This would be the necessary outcome of broadening a concept to its utmost possible limits. The name "Scott" would then signify anything which could be said about the individual, Scott; so that any judgment into which "Scott" entered would become analytic and trivial in Mr. Russell's sense.

X

Just as a judgment may be analytic in form and synthetic in effect, so a judgment may be synthetic in form and analytic in effect. Judgments such as "x has the relation R to y," where R is not identity, or "x has the quality Q," are, in general, taken to be synthetic, to add something to their subjects through their predicates. But if the meaning of x already includes the relation R to y or the quality Q, the judgment is synthetic in form only. Its effect is analytic. Thus Kant's example of an analytic judgment, *viz.*, "body is extended," is synthetic in form but analytic in effect because the attribute of extension is by definition included in the meaning of the term "body." If I mean by "Scott," among other things, "the author of *Waverley*," then the seemingly synthetic proposition, "Scott wrote *Waverley*," is analytic in effect.

Whether the predicate of a judgment adds something to what is meant by the subject depends, therefore, on what one means by the subject. This is why "Scott is the author of *Waverley*" or any similar statement of identity can convey more to the mind than "Scott is Scott," and why "body is extended" or "Scott wrote *Waverley*" can convey no more than "body is body" or "the author of *Waverley* wrote *Waverley*."

It is only because we do not know (analytically) what we can mean by a proper name or single word, or because we use them with a deliberately restricted meaning, that any judgments are *synthetic in effect.* Ignorance is the great restricter of meanings, and ignorance renders certain properties external to objects, which on different definitions of these objects, become internal to them. One usually means by a word something less than an object with all its possible predicates and relations, and so the object can be viewed as entering into new relations and acquiring new predicates. "Napoleon conquered Europe"

shows Napoleon in a new rôle, and is a synthetic judgment (both in form and in effect), if one does not include "conqueror of Europe" in the meaning of the name "Napoleon." But this is an arbitrary or accidental restriction of the meaning.

What has been said of the analysis of the meaning of proper names is true also of single words which stand for universals. A universal can be described, or it can be represented by a single word. The description means the same thing as the single word, but means this analytically, while the word means it unanalytically. The word is capable of being extended or contracted in meaning to take in more or fewer of the predicates and relations of the universal meant. A word which signifies a universal may enter in judgments of synthetic or analytic form, and these judgments will be synthetic or analytic in effect according to the interpretation put on the word.

That single symbols (proper names in a generalized sense) can mean the same objects as descriptions, and that the statement of such identities of meaning is highly fruitful for knowledge, rather than trivial, are, then, the consequences of this view of description as analysis. It follows, also, that proper names (in the more restricted sense of this term), if they mean bare individuals, are of no more use to thought than x's and y's whose values are not known. And even when their meanings are wider than such references to bare individuals, proper names give us no more determinate, or invariable, knowledge of individuals than do descriptions. Whatever can be referred to through invariable symbols is universal. Aristotle himself, to whom individual substances were the sole realities (God only being excepted), believed that universals alone were the objects of *scientific* knowledge. What can be said of individuals must be said in universal terms, and only when reference to individuals is dropped is knowledge completely freed from variability of meaning. Completely determinate knowledge is of universals

and their connections. Hence perceptual knowledge is never completely determinate, never wholly compressed into the concepts of invariable meaning which function in perception; and yet it is the only concrete knowledge, that is, the only knowledge in which the individual merges with the universal. Complete conceptual determination and concreteness do not go together; what is completely determined for conception is never concrete, and what is concrete is never completely determined in concepts.

XI

Knowledge in universal terms tends to become analytic in form, for here, as elsewhere, concepts enlarge themselves by including more and more of the predicates and relations of the universal meant. To the layman, water is something wet, colorless, tasteless, and odorless; if he were asked to define it, he would describe it in this way. But if he studies chemistry, he finds that water is made up of hydrogen and oxygen in definite proportions, and his concept must be enlarged by the judgment that "water is H_2O," a judgment of analytic form but synthetic effect. He will discover that water boils at 212° Fahrenheit under atmospheric pressure, and at lower temperatures under lower pressures, etc.; and these facts, together with countless others, must be included in his concept. In the end his concept of water will completely sum up his knowledge of it. The word "water" will mean all that water is known to be and do, and a judgment about water, *e.g.*, that water boils at 212° Fahrenheit under atmospheric pressure, will tell him no more than is included in his concept. It will be an analysis of this concept.

Thus the chemist, if he discovers a property or law which is "external to" water as he conceives it, will straightway take this property or law into the meaning of the term. But in addition, there will always remain a perceptual "somewhat" to which the

name "water" belongs, and this "somewhat" will be spread out in numberless individual instances. But the evasive element of individuality will never be a part of the meaning of the universal term. The concept will include much that can be known only by inference from the perceptual properties of water; these inferred properties, *e.g.*, molecular and atomic structure, etc., will also be described in universal and invariable terms.

A science becomes analytic in form for this reason: it tends so to broaden its concepts of a subject-matter, originally given in perception, that an analysis of what is meant by these concepts states the laws of the science. The subject-matter is defined as that which follows the laws; anything which does not obey the laws is not included in the scope of the science. Thus matter, originally in perception something solid and extended, becomes anything that conforms to a certain set of principles — physical principles; and mind becomes anything that conforms to another set of principles — psychical principles.

The drift toward the analytic form is especially evident in the mathematical sciences. Crude perceptions of a spread-out and diversified something called "space" are enlarged by the discovery of principles which can be exactly stated in universal terms, and which can be interpreted as referring to these perceptions. "Space" is finally conceived as that system of objects which obey geometrical axioms and postulates; and, when space is thus defined, to say that space conforms to geometrical laws is to say merely that "space is space." If different sets of axioms and postulates which can be equally well interpreted by the original experience are discovered, these axioms and postulates will define different kinds of space — Euclidean, and the numerous varieties of non-Euclidean space. Thus it becomes a trivial and analytic statement to say, *e.g.*, that the parallel postulate holds for *Euclidean* space, for Euclidean space is defined through this postulate and others. But to say that this postulate

holds for space *as such* is to raise the question as to whether the Euclidean analysis of space is the true analysis, and so to force a choice between this type of geometry and other types. The judgment, "Space *as such* is Euclidean space," though analytic in form, is not a trivial statement of identity. It purports to add something to knowledge by showing that what we originally meant by "space" is truly analyzed by the Euclidean axioms and postulates. This judgment is synthetic in effect and can be proved or disproved by experience.

A science which assumes the analytic form does not lose its experiential content, and this is no less true of the mathematical sciences than of the physical, biological, or psychological sciences. Number and space, though they may be defined through the laws of arithmetic and geometry, are still objects (universals) met with in experience. A "science" which had no point of application in presented objects would not be a science, but an uninterpreted or partially interpreted system of symbols (or concepts). The description of the subject-matter given in the laws, the postulates and axioms, must be anchored in experience; the variable terms, the x's and y's, described must mean something apart from the descriptions in which they appear. Thus if Euclidean space is simply the x which the postulates of Euclidean geometry describe, one does not know fully what this x is. There may be many different objects, *e.g.*, certain series of numbers, etc., which are possible values of this x. These postulates become the laws of space only when they are interpreted by the spread-out and diversified something called "space." The only reason why one can speak of the postulates of Euclidean or non-Euclidean geometry as postulates of geometry is that they are rooted in an original experience of space.

An uninterpreted or partially interpreted system of concepts is merely the possibility of a science. It becomes an actual science only when it is taken to be a description (or an analysis) of

some field of presented objects. The fact therefore that a science tends to become analytic in form does not free it from the necessity of binding itself to experience by a synthetic judgment — a judgment which conveys the information that the subject-matter as perceived *is* the subject-matter as described.

XII

A second argument by which Mr. Russell supports his theory that descriptive phrases are without meaning excepting as they occur in a context, brings up the problem of "reference to the non-existent," that is, of how symbols can be used significantly when there is no object which they mean.

"Suppose we say: 'The round square does not exist.' It seems plain," says Mr. Russell, "that this is a true proposition, yet we cannot regard it as denying the existence of a certain object called 'the round square.' For if there were such an object, it would exist: we cannot first assume that there is a certain object, and then proceed to deny that there is such an object. Whenever the grammatical subject of a proposition can be supposed to be meaningless without rendering the proposition meaningless, it is plain that the grammatical subject is not a proper name, that is, not a name directly representing some object. Thus in all such cases the proposition must be capable of being so analyzed that what was the grammatical subject shall have disappeared." [1] And this argument is carried still further: the *Principia Mathematica* asserts that a function (a predicate) which is universally true may nevertheless not be true of an object described as "the so-and-so." Though it is true of every object that it is an identity, it is not true of "the round square" that it is an identity. This object is therefore nothing.

But if the round square is nothing, does it follow that "the round square" is without meaning? Are not the two phenomena

[1] Whitehead and Russell, *Principia Mathematica*, i, 69.

of significance, to be without meaning and to mean something non-existent, different phenomena?

There is an air of contradiction in the idea of meaning or referring to the non-existent. But this semblance of contradiction disappears if "reference to the non-existent" is more accurately described as *the significant use of symbols where there is no object for which they stand.* It is possible to employ symbols significantly (with syntactical significance) when there is no object to which the group as a whole refers, and this is what Mr. Russell denies or overlooks. But even if it were true that a phrase which "means a non-entity" is without meaning, it would not follow that because some descriptions mean non-entities, all descriptions mean non-entities and are therefore without meaning. If "the round square" is a meaningless expression because there are no round squares, this fact does not imply that "the President of the United States" is meaningless because it is a descriptive phrase of the same form as "the round square."

However, the important question is — Can a description, or a symbolic group, be significant and yet not refer to an object?

The *Principia Mathematica,* being concerned with logic rather than epistemology, does not analyze the concept of *meaning.* But certain assumptions as to the nature of meaning seem to be implicit in its earlier, more strictly philosophical sections. So far as we can disentangle these assumptions, the view of meaning seems to be this: that groups of symbols which state propositions, and proper names (in a generalized sense), are the only symbols which have meaning in themselves. Thus descriptive phrases fall between two stools: they are groups of symbols, but they do not state propositions; they play a part like that of proper names, yet they are not proper names. If this is the assumption of the argument which rejects descriptive phrases as meaningless outside a context, the conclusion must be granted. It then becomes necessary to translate descriptive phrases into

another form, that is, to treat them as significant only when they occur in propositions, and to construe these propositions as statements which make use only of proper names (in the generalized sense). But the possibility that groups of symbols which do not state propositions, and yet are not proper names, might be significant in their own right is not considered. And this is what we have been urging all along: that there is some legitimate sense of the term *meaning* other than those senses which appear to be assumed in the *Principia Mathematica.* This type of meaning—syntactical significance—covers both propositions and descriptive phrases; and just as a group of symbols which states a proposition may as a whole refer to no object, so a descriptive phrase may as a whole refer to no object and yet be significant.

While proper names and single words, that is, simple symbols, unless they are defined through symbolic groups, arouse intentions directed toward objects — intentions which depend on the existence of these objects — a symbolic group, on the contrary, arouses an intention which is a function only of subordinate intentions and of the structure of the group. The meaning of a symbolic complex does not depend on the existence of an object for which it stands as a whole. In order that a symbolic group may be significant, it is necessary only that its constituents be significant (or be definable in terms of significant complexes) and that it have a form. It is in this sense of *meaning* that "France loves Germany" has meaning. There is no object, *France loving Germany*, but the phrase is significant because its constituents are significant and grouped according to a plan of logical structure.

To say that a description "means a non-existent object" is a convenient but inaccurate way of speaking, for a "non-existent object" would not be an object and could not be referred to.

The clearer manner of statement is that a description can be significant apart from any reference, as a whole, to an object. "The round square" means roundness and squareness attaching to a term which is symbolized by a variable; it means "x as modified by roundness and squareness." "The king of France" means "y as modified by the kingship of France." And whether or not any such unities of terms and predicates exist to be referred to is beside the point. Mr. Russell's argument must be countered by a broader conception of meaning.

It is because the existence or non-existence of an object signified does not affect the significance of a symbolic group that "to mean a non-entity" is not to be meaningless. Only simple symbols which refer directly to objects require the existence of an object meant. Such symbols mean "categorically," but syntactical or group meaning is "hypothetical," for it presupposes no object corresponding to the group as a whole.

To assert that "the round square does not exist" is not, then, to assume that there is an object called "the round square" in order to deny its existence. Nor does the truth of this assertion lead to the conclusion that the phrase "the round square" is in itself meaningless.[1]

[1] As for the statement of the *Principia Mathematica*, i, 87, that the law of identity does not apply to descriptions, *i. e.*, that one cannot say "the round square is identical with the round square," this depends on what the assertion of identity means. It has been shown above that $x = x$, as a general principle of symbolism, means that the symbol x is always to be interpreted in the same way, whether or not it stands for an existent object. Thus "the round square = the round square" does not necessarily assert the existence of this object, but it does assert that the significance given to this phrase must be the same in all contexts; that "the round square" can be substituted for "the round square" in any discourse, without an alteration of the meaning. But if "the round square is identical with the round square" is interpreted existentially, to mean that "the round square has identity," this will not be true unless the round square exists. However, the symbolic interpretation of the law of identity is separable from the existential interpretation: "x is x" need not be construed as predicating identity of an object meant by x. See above, ch. II, sec. vii.

XIII

The particles "all," "every," and "each" introduce a new idea: that of classes.

"All men," "every man," "each man," are different ways of referring to the class *men*. As in the case of the other particles, these particles have no meaning in themselves; they must appear in connection with symbols that refer to universals or complex predicates. They do not signify constituents of the fact represented by a phrase in which they occur; they are signs of interpretation. Just as "the" tells us that one and only one instance of a universal is meant (though this instance is indeterminate), and "a" that one among many possible instances is meant, so "all" tells us that the universal is signified in a certain manner of its occurrence — in its occurrence in a class.

Classification rests on universals. Though a universal is something distinct from any or all of its instances, though it is an aspect which can be singled out and made an object of reference, nevertheless, a universal occurs in instances, it modifies a number of terms; and a class is the universal taken in conjunction with the terms it modifies. It is a plurality of terms modified by the same universal. Any predicate determines a class — the class of the terms to which this predicate attaches. A *relation* determines a number of classes: "*A* hates *B*" is, for example, a case of the class of "hatreds"; but from this relation the class of "those who hate *B*" and the class of "those whom *A* hates" can also be derived. "*A* is between *b* and *c*" is a case of the class of "relations of betweenness," but it also determines a class of "those objects which are between *b* and *c*" and "those objects between which *a* occurs." In every case a class is a plurality of terms modified by a single quality or relation.[1]

But a class is at once a plurality and a unity. It derives its

[1] A class of one member is the only exception; this is not a plurality of members: but if it is a class, it is a *totality* of *one*.

unity from the presence of a predicate in a number of distinct terms, and its plurality from the distinctness of the terms. The difficulty of conceiving a class as an object, says the *Principia Mathematica*, is connected with the "ancient dilemma of the One and the Many." "If there is any such object as a class, it must be in some sense *one* object. Yet it is only of classes that *many* can be predicated. Hence, if we admit classes as objects, we must suppose that the same object can be both one and many, which seems impossible." [1] But the impossibility is less real than apparent. It is no more present in the conception of a class than in the conception of any object which is analyzed into constituents, broken up into parts which form a whole. Every analyzable object is both one and many, a unity of diverse elements; and if one balks at the problem of the One and Many, he must reject all analysis.

The unity of a class, however, is different from the unity of a fact (or a *group*), and for this reason a class, though it is none the less an object, is not the same kind of object as a fact. It is an object of a different order. "Caesar loving Brutus," "Boston's being near to but north of New York," are expressions that stand for factual groups. In these groups the universal unites the terms so that (taken with the universal) they become a single instance of this universal. The predicate which determines a class unites the members in no such way as this. (But if there are classes there will also be factual groups, and if there are factual groups there will also be classes.) The unity of a class is of the peculiar kind called *totality*, and this is a new primitive idea — the defining idea of classes.

A class is a universal *in the totality of its instances*, that is, it is an object like "man" as it appears in this sort of unity with diverse terms. "Man" means a universal apart from its instances, that is, in abstraction; but *man* is not only separable

[1] Whitehead and Russell, *op. cit.*, i, 75.

from its instances, it is also joined to them, spread throughout them. "Men" as distinguished from "man" means the universal as it is particularized in a multitude of different terms. This object is *one*, and can be referred to through a single symbol, because the universal gives it the unity of totality, though this is a "looser" unity than that of a single fact of diverse terms. No part or selection of the terms to which a universal attaches is a class, a totality, through the presence of that universal alone. Every universal determines one and only one totality. Thus a portion of the class *men* would be a class only through the presence of some other predicate than *man*, *e.g.*, the predicate, *brave man*, or *strong man*. A class therefore is nothing short of a universal in its concrete entirety. The universal, in making the instances *one*, makes them a totality, but at the same time permits them to be distinguished as instances.

It follows from this notion that a class is not a sum of objects. The expression, "*a* and *b* and *c*, etc.," does not designate a class, for objects which do not belong to the same class can be thus added together. A mere summation of objects will not give us a class predicate which determines them to be a totality. "The table and the chair and the book, etc.," are not a class through the fact of being conjoined; they become a class only if there is some predicate which gives them the unity of totality. If they are "all the objects in this room" or "all the objects I now see," they are a class. Nor does the similarity of objects alone make them a class, although objects of the same class are similar. Only if x and y and z, etc., are "all the objects similar to a given object c," do they form a class, and in such cases this class will be a totality determined by a predicate.

Classes are cross-sections of the world of fact. Only very limited classes are presented in their wholeness; most classes can be known only conceptually or symbolically, that is, as *the* (x) totality determined by such and such a predicate. Though "all

men" means *one* object, no one has known *all men;* and thus classes seem not to be objects, but constructions of the mind. Yet the inability of knowledge to grasp most classes as totalities in perception does not disprove the objectivity of classes. The idea of totality once being given, — as it is in the restricted totalities of presentation, — this idea can be extended to totalities that lie beyond presentation, and to infinite totalities.

"Each" and "every" as well as "all" precede expressions that refer to classes, but the distinctions between "each," "every," and "all" are difficult to draw. The difference seems to be one of emphasis. "All men are sinful" appears to emphasize the fact that the class of men is a totality. "Every man is sinful" means the same, that is, that all men are sinful, but emphasizes the plurality of the class. "Each man is sinful" also emphasizes the plurality of the class, but at the same time seems to include a reference to an enumeration of the members. But whatever the shades of difference in the meanings of "each," "every," and "all" may be, these three words indicate manners of reference to classes.

A reference to a class is clearly a description of the form "the so-and-so" rather than "a or some so-and-so." The totality is meant through an undetermined x which has only one possible value, that is, the totality in question. "All persons" means "the (x) totality determined by the predicate 'person'"; this expression is not susceptible of many interpretations as is "a person." [1]

XIV

The discussion of description and analysis has brought out the following points:

[1] It is plain that a class could have one member or an infinity of members and still be a totality. "All the numbers which, when added to a number give that number," designates a totality of one member — the number zero. "All the objects which can be put into one-to-one correspondence with the natural numbers" designates an infinite totality.

The line of cleavage between proper names or single symbols and descriptive phrases is determined by structure alone; the distinction is not that proper names (in a generalized sense) have meaning in themselves, while descriptive phrases have meaning only in a context. Description is analysis, for the predicates and relations of a term are logical "parts" of it, and what is represented analytically by a descriptive phrase is represented unanalytically by a proper name or a single symbol. This agrees with common usage, which treats proper names as equivalent in meaning to descriptive phrases. Syntactical significance is present in all symbolic groups, and only when this type of meaning is neglected (or confined to groups which state propositions) does the contrary view, that descriptions have no meaning in themselves, become plausible.

Symbolic groups are significant apart from the existence or non-existence of objects meant, for their meaning as a whole is a function of the meanings of their parts and their logical form, and nothing else. This wider notion of meaning covers that of descriptions outside a context. If there is no object corresponding to a description, this fact does not deprive the description of its syntactical significance. "To mean a non-entity," that is, to be significant when there is no object signified, is not to be meaningless.

A description signifies a term, together with a predicate or complex of predicates attached to this term; but the term is represented by a variable which, in language, is understood to be present through the sign of interpretation (the particle) preceding the phrase. Descriptions are of the general form (xR): the object has this structure and the symbols themselves take on this structure if the variable, which is understood, is supplied. Thus "a man" means "a human x," "some friend" means "some friendly y," "the president" means "the presiding z." The term described becomes the center of a complex of char-

acters and relations, and the variable through which this term is signified is to be interpreted in the sense indicated by the particle. "The" means that the variable has only one possible value. "A" and "an" mean that it has many possible values; "any," that the value is to be chosen at random from among these many possible values; and "some," that it is to be chosen according to an unspecified condition of selection.

Though a description does not *state* a fact, it always has reference to a fact — to the fact through which the object in question is described. "The president of the United States" refers to the fact that "someone presides over the United States"; "a bard of the sixteenth century," to the fact that "someone wrote poetry in the sixteenth century"; "a man," to the fact that "someone is human." And whether the description signifies an existent object, or refers to no object, will depend on the truth or falsity of the propositions which assert these facts. This is the basis of Mr. Russell's reduction of descriptions to the form of propositions which make assertions about terms.

But it is not necessary to translate the description into another form to observe that it refers to a fact as well as to a term. The term is, as it were, viewed through the fact; the fact is viewed as centering about the term, so that the term assumes a pivotal rather than a subordinate place in the fact. (This is symbolized by writing the x both inside and outside the group through which it is described. Thus $(x((\mathrm{x}R)Sa))$ indicates that the x is described through the whole complex $((\mathrm{x}R)Sa)$.) This centralization of the fact about a single term is the only difference between the fact as it appears in the description and as it might appear in some other (non-descriptive) form of expression. If a description is asserted, — and it follows from this view that descriptions *can* be asserted, — it will be seen that the description may state a fact, the fact on which the description is based. "The president of the United States is," affirms the same

fact as "someone presides over the United States"; "the author of *Waverley* is," makes the same assertion as "someone wrote *Waverley*," where "someone" is taken to have only one possible value. "A man is" affirms no more nor less than "someone is human." [1] The difference in these assertions is one of form only, and not of content. Therefore descriptive phrases are not distinguished from "propositions" through the fact that the latter mean complexes of terms and predicates or relations, while the former mean only terms. Both stand for complex wholes (if they stand for any objects); both mean facts.

Variable significance, which is always present in descriptions, is a distinct kind of meaning, different from syntactical meaning or from direct reference to objects through simple symbols. The variable, meaningless in itself, is construed as a symbol because it enters as a constituent in a significant whole; it is a symbol which arouses no specific intention, but which must be treated as symbolic because of its setting. And if it is given a value this can be, by the principle of identity, one and only one value. The variable is no more open to an equivocal interpretation than any other symbol. A variable may, however, have many possible values, and it will then be ambiguous — a means of attaining generality of reference.

Both universals and individuals can be described, and both are variably (indeterminately) signified by descriptions. But an individual is still variably signified when it is given a proper name, for the proper name must be defined through a description, which contains a variable element; otherwise the name would stand merely for a "this" of presentation and would be even less determinate in meaning than if it were defined through a description.

The only data of perception which can be represented by

[1] "Is" as a sign of assertion in these expressions adds or subtracts nothing from the meaning of the phrase.

invariable symbols are universals, and so the only knowledge which is completely free from variability of meaning is knowledge in universal terms, that is, abstract statements, laws, which make no reference to individuals.

Definitions are analytic in form. They are statements of identity, such as $a = (xR)$; and they can be reduced to the form $a = a$. But they are not on this account trivial, since their effect is synthetic; they add to the knowledge of the object meant, a knowledge that it can be analyzed as (xR), or that predicates which were originally external to the meaning of a can be included in this meaning. Concepts of restricted meaning enlarge themselves through such statements of identity by taking in more and more of the predicates of the object meant, and in this way knowledge tends to become analytic in form. It tends to sum up all that can be said of an object in the very concept (the very name) of the object. But the postulates and axioms of a science do not completely determine the subject-matter of the science, even though an analysis of what is meant by "the subject-matter" may be a statement of the laws of the science. Knowledge of analytic form must be anchored to experience by a judgment which is synthetic in effect — which asserts that the objects within a certain field of experience *are* the objects thus analyzed. Euclidean or non-Euclidean geometry is geometry because its postulates can be interpreted in terms of the experience called "space."

Lastly, there is a type of object — a class — which is referred to by descriptions prefaced by "all," "every," and "each." Classes are not groups, not wholes of the form $(Rab \ldots)$, but a class is nevertheless *one* through the presence of a universal in terms. Its unity is of the undefined sort known as *totality*. Classes, with the exception of the limited totalities given in perception, are known only conceptually, that is, through symbols, as "the (x) totality determined by such-and-such a predicate."

Descriptions, then, do not stand apart from other symbolic expressions. All symbolic groups have significance in themselves, and this significance is independent of the existence of an object meant. The possibility of using symbols significantly, either in descriptions or in propositions which are not descriptions, when no "real" object is referred to, lies at the basis of the distinction between truth and falsity. Meaning becomes truth when it is joined to existence; it becomes falsity when it is severed from existence; but without meaning there is neither truth nor falsity.

CHAPTER V

TRUTH AND FALSITY

I

TRUTH and falsity are properties of symbols. A symbol is true if it stands for an object; it is false if it is significant, yet stands for no object; or, in the words of Thomas Hobbes, "True and false are attributes of *speech*, not of *things* . . . truth is the right-ordering of names." [1] If the term "speech" is widely enough construed, this definition of the aim of knowledge is a corollary of the theory of meaning that has been presented. The "right-ordering of names" is building symbols into structures that correspond to structures of fact, and this correspondence is truth.

Belief and disbelief do not alter the truth or falsity of symbolic expressions, for the existence of meaning is a sufficient as well as a necessary condition of the existence of truth. No one can be in error unless what he believes has meaning, and he cannot be convicted of error if his meaning is misunderstood; nor can he believe truly unless that which he believes is significant. Yet truth and falsity are independent of belief; any idea that can be entertained is either true or false. Once a meaning is fixed, its truth or falsity is fixed; and meanings are carried only in symbols. This is the idea behind Hobbes's definition. Yet it must not be forgotten that a symbol is more than a mark, a sound, a gesture, or an image; it is any of these together with the effect it has in a mind, that is, together with the psychical attitude to which it gives birth. Symbols are concepts, and to say that truth is a property of symbols is to say that it is a property of concepts.

[1] T. Hobbes, *Leviathan*, Part I, ch. 4.

Truth, moreover, cannot be defined without some reference to reality or existence; ideas, thoughts, perceptions, are true when they present or represent what *is*. To apprehend a truth is to apprehend the existence of something meant. Thus knowledge from the outset is directed toward reality, and finally toward a metaphysical goal. Epistemological questions project themselves toward metaphysics through the concept of truth. But one must approach metaphysics humbly. He must begin with a limited concept of the real, which he may alter or abandon in the light of further criticism; for a theory of the relation of knowledge to reality can have no basis except in an analysis of knowledge as a phenomenon, and this analysis leads to a limited notion of reality and of truth.

The definition here set forth is of this sort. Truth as reference through symbols to existent objects may be a reference to objects that are, metaphysically speaking, only "appearances"; or if these objects are not appearances, they may be objects that are not separable from knowledge, they may be mind-objects only. And there are many other metaphysical contingencies which the definition ignores — to which it is, in Mr. Whitehead's phrase, "closed." [1] It adopts a restricted concept of existence, and this yields a restricted concept of truth, whose immediate claim to acceptance is that it is workable in science, mathematics, and every-day thought, whether or not it is finally able to withstand metaphysical scrutiny.

II

The examination of this definition — of the notion of existence in terms of which it is stated, together with the tests of truth to which it gives rise — can be better understood if it is prefaced by an inquiry into another view of truth: the view that truth is not a property of symbols, but of entities which are

[1] A. N. Whitehead, *The Concept of Nature* (1920), ch. 1.

neither symbols nor existing objects, yet which have a being in themselves.

On this theory, it is to "propositions" that truth attaches, and a proposition is not thought of as a symbol or symbolic group coupled with the psychical set or intention it arouses in a mind. A proposition is independent of psychical processes; it is a meaning, but a meaning considered apart from its setting in a mind. It is something distinct from symbols, to which they refer, and yet it is not an object. The proposition thus conceived enters as a *tertium quid* between the symbols, in which it is conveyed, and the datum or object, of which it is true. It is a wedge which opens the way for a new class of entities — subsistent entities — which are to be distinguished from objects. The peculiar manner in which these subsistents are apprehended is said to be *conception;* to conceive is to know something, to refer to something, but not to refer to an object. On this view, the initial reference of conception is to a subsistent proposition, and the proposition in its turn may or may not be directed toward an object. If the proposition is directed toward an object, if the *tertium quid* meant by the symbols exists as well as subsists, it is true; otherwise it is false.

This theory purifies truth of all psychical elements. Truth and falsity are not determined in any sense by the act of thought; they belong only to these independent entities, which can be thought of but are not created by thinking. At the same time, truth and falsity are not taken to be properties of objects, so that the difficulties of one type of view, which says that there are "false objects," are avoided.

The most convincing arguments for the assumption of these subsistent entities are derived from the consideration of false and negative propositions. Mr. G. E. Moore puts the case for them in the following way: "How can a thing 'appear' or be 'thought of' unless it is there to appear or be thought of? To say

that it appears or is thought of, and yet that there is no such thing, is plainly self-contradictory. A thing cannot have a property unless it is there to have it. . . . When I think of a unicorn, what I am thinking of is certainly not nothing; if it were nothing then, when I think of a griffin, I should also be thinking of nothing, and there would be no difference between thinking of a griffin and thinking of a unicorn. But there certainly is a difference; and what can the difference be except that in the one case what I am thinking of is a unicorn, and in the other a griffin? And if the unicorn is what I am thinking of, then there certainly must *be* a unicorn, in spite of the fact that unicorns are unreal. In other words, though in one sense of the word there certainly *are* no unicorns — the sense, namely, in which to assert that there are would be equivalent to asserting that unicorns are real — yet there *must* be *some* other sense in which there are such things; since, if there were not, we could not think of them." [1]

The status of propositions, on the view in question, is that of Mr. Moore's unicorns and griffins; they are entities which are "there" in some sense, though plainly they are not "there" in another. They are *objectives*, which may be real or unreal; which *subsist* rather than exist.[2]

Subsistence is a kind of being to which existence may be added, but to which existence is not necessary. In the realm of subsistence, the lion and the unicorn lie down together. To think of a lion or of a unicorn is to think of entities that partake equally of this impartial being, this thinner reality, which includes the possible and the imaginary as well as the real.

[1] G. E. Moore, "The Conception of Reality," in *Philosophical Studies* (1922), p. 215.

[2] The terminology is that of A. Meinong's *gegenstandstheorie*. Two among Meinong's important works on this subject are: *Untersuchungen zur Gegenstandstheorie und Psychologie* (1904, Leipzig) (a collection of studies by several authors including Meinong), and Meinong's *Über Annahmen* (1902, Leipzig).

III

The concept of the "objective" springs from what appears to be a logical necessity in the analysis of meaning. Meaning must have an objective reference, and it is thought therefore that there must be a thing meant wherever there is significance. If meaning is reference to something — a relation in which the mind is only one term — unless this relation is to disappear, it must terminate in a referent. This is the idea behind Mr. Moore's statement that "to say that (a thing) . . . is thought of, and yet that there is no such thing is plainly self-contradictory." When the referent is a non-entity, it cannot (if the argument is pursued) cease to be a referent; it must therefore have some status. It is not nothing, for if it were the meaning relation would collapse, being deprived of one of its terms.

Furthermore, the reference of symbols is not merely to psychical states, to images, to the content of the mind. The very essence of the meaning activity is that through it the mind reaches out toward something other than its own states;[1] and yet, if this "something" is to be grasped, it must be somehow given. It must have a being which allows it to be referred to when it is, in some sense, not a presented or existent content of the mind. It must be "there" in some way and not "there" in another; otherwise meaning loses its objectivity of reference and becomes intra-psychical in the narrowest meaning of the term.

But the dilemma here is not so sharp as it appears. Objectivity of reference does not require the assumption of subsistent (and sometimes unreal) entities as the second term of the meaning relation. The reference of a symbol, either simple or complex, can be directed beyond the mind's content when the entity referred to is in no way presented or "there"; and a complex

[1] That is, toward something other than its own present states. This does not preclude the metaphysical hypothesis that all objects referred to are ultimately "mental" in a wide sense of the term.

expression can have an objective reference when there is no object or objective to which the expression as a whole refers.

Consider, first, the meaning of simple symbols which stand directly for objects. The theory of objectives assumes that these symbols, no less than complex expressions, have significance through reference to subsistent entities. It is not sufficient that the meaning relation shall have been grounded in an existent term; it must continue to be grounded in a subsistent one. Now, if to mean something is to intend it, to take an attitude of mind appropriate to it, the continued being in any sense — either as a subsistent or existent entity — of that which is meant is not necessary to the meaning. Indeed, anticipating or intending is just the sort of reference to objects that is compatible with the absence, as well as the presence, of what is referred to. Meaning is not a static relation between the mind and any sort of entity. It is an activity, and this activity has a direction away from the mind's present content, whether or not there is a given object (or objective) in which it terminates. The direction of a meaning is determined once an object *has been* the terminus of the activity; the activity continues to intend, to be appropriate to this object thereafter, when the object is absent or non-existent. It is sufficient therefore that the things which simple symbols mean *shall have* existed; the subsequent use of these symbols does not demand the persistence of their referents, either as subsistent objectives or existent objects. A meaning can be carried by an intention alone, and the reference does not lose its objective direction.

A second type of meaning is that of complex expressions. If the mechanism of conception is analyzed (as it has been) into the grouping of symbols, the groups as a whole continue to refer, to mean — and to mean something other than a present psychical content — whether or not there exists or subsists an object or objective to which they refer. The meaning of a sym-

bolic group has been shown to be a function of other meanings, together with a scheme of logical structure. A number of subordinate meanings enter into a unity to form a new meaning, which cannot be described as a simple "mind-referring-to-object-or-objective," as Mr. Moore would describe it; for the situation is made complex by a new factor — a plan of structure by which the symbolic elements are welded into a whole. If there is an object referred to, the reference is indirect. The symbolic group is a construction which is not taken, in its entirety, to stand for an object as a single word or proper name might be. From the direct references of the elements to things that exist or have existed, the mind fashions a secondary reference — a reference composed of references. Thus I build up the group, "the snow will melt to-morrow"; I join the several direct references of the words, whose meanings are determined by previous use, into a single indirect reference of a certain form; but I can know only to-morrow whether this secondary reference does or does not terminate, as a whole, in an object.

If there is an object corresponding to the whole expression, this object will be both a unity and a plurality, a whole of parts; *e.g.*, the unified fact, "the melting of the snow on the morrow." But if there is no such unified object, the parts will still exist (or will have existed) as a plurality; *e.g.*, there will be a to-morrow, and there have been snow and melting. Therefore the meaning of the group will be grounded in a reference to objects. The significance of the expression will be directed toward something other than the present content of the mind, despite the fact that it corresponds in its entirety to nothing.

It is in this sense that the false statement, "Shakespeare wrote *The Critique of Pure Reason*, has an objective reference. "Shakespeare," "wrote," "*The Critique of Pure Reason*," stand for objects. These references to existent things, or to things that have existed, lend objectivity of reference to the whole. They

give it a foundation in fact, though it corresponds to no fact. And since the symbolic elements of any significant group must stand directly for objects which exist (or have existed), or must be definable in terms of symbols which stand thus for objects, no symbolic group is without an existing *locus* of reference of the sort that the primary, direct act of meaning demands. The meaning of the group cannot collapse into a total absence of relation to the objective world, for the objects meant by the elements of the group are in this world and thus determine the mind to have a direction beyond itself.

This is not all. The plan of structure is no less objective than the constituents meant by the simple symbols, even if there is no fact which combines these constituents in this structure. The logical form intended is presented, and exists, in the symbolic expression as a universal of which the group is itself an instance. The structure meant is identical with the structure of the concept, that is, of the symbols; and it is equally as real, equally as objective, in its conceptual or symbolic instance as it would be in a non-conceptual or non-symbolic instance, for it is the same in both.

The fixity of complex concepts arises from their logical form. Given a certain form in certain symbols whose individual meanings are determined, and the meaning of the concept is determined. If there is an object whose elements are the ones meant by the simple symbols, and whose structure is identical with the structure devised by the mind for this group, this object will be the thing meant. But there need be no such object; the meaning is still fixed and still turned toward the world of objects. The mind, once it invents a concept, is bound by it. The concept seems to become external to thought, to pass over into a realm beyond the mind's caprice; for the form of the concept, though it is constructed in thought, is a universal which may appear elsewhere. The constructive activity of the mind is exercised in a

medium which is not purely of the mind — in the medium of logical form. Instead, therefore, of saying that thought arbitrarily assigns meanings to complex concepts, as it does to single words or signs, one can say that these concepts assign meanings to thought. The logical structure of the concept reaches out toward objects and, as it were, selects the one (if there is one) to which the concept corresponds. The structure of the concept means the structure of the fact because it is identical with the latter. And so the verification of a complex expression is a search in the direction of the objects meant by the constituents of the expression for an instance of the same logical form which is present in the expression. Thus the mind invents its concept but discovers their truth or falsity.

IV

The *tertium quid* which, on the theory of subsistence, is assumed to intervene between symbols and their objects disappears in this analysis of meaning. It is absorbed in the symbols themselves. Meaning is not, as the theory of objectives would have it, what is referred to through symbols; it is the referring itself, that is, the mean*ing* rather than the meant. And this act is always turned in the direction of existing objects, even though there may be no single object in which it finds its fulfillment; so that meaning does not become purely a relation of one psychical event to another. It is an act in which psychical events point beyond themselves, and symbols are the perceptible embodiments of this act.

Not only are symbols the instruments of knowledge, they are of its very stuff. Mediate knowledge and a large part of immediate knowledge are words, images, signs, taken with the effects of these in minds. There is no independent thing called "knowledge," which words, images, psychical attitudes, signs signify. The act of knowing itself, in so far as it is not pure awareness, is

the use of symbols. To assume that there is some such thing as the "proposition" or "objective," which is the referent of symbols but which is itself neither a symbol nor an object, is to postulate a detached element of "meaning" as the terminus of thought. All attempts to catch this elusive and independent "meaning" end in the capture of images, of words, of attitudes or sets of the mind. The view that propositions are what symbols signify, that they are meanings apart from symbols and from the mind in which these symbols have significance, cuts off meaning and hypostatizes it, as if it might have being in itself; but this is as impossible as that a picture might exist without the canvas on which it is painted.

The elimination of these subsistent entities subserves the end of economy of assumption. Truth is to be found in the two-term relation between symbols and objects; the proposition as a third entity is not needed. Symbols either signify existent objects or no objects, for the phrase "existent object" is a tautology, and the phrase "non-existent object" is a contradiction. The category of objectivity is not wider than the category of existence; there are no non-existent entities of any sort. And yet it is possible to speak or think significantly without assuming an object thought or spoken of.

It is this notion, that a concept need not have an object, which supports Kant's refutation of the ontological proof of God and makes ontological arguments in general invalid. No concept implies the existence of an object conceived. But the theory of subsistence restores the ontological argument in a rarefied form. Though one cannot, on this theory, argue from a concept to the existence of an object, he can always argue to the subsistence of an objective. "A thing must *be* in some sense to be thought of." This gives every concept a referent with a being — a being which is often non-existence. A strange multiplication of ontological types! One sort of being should suffice. If a thing

does not exist it has no being; to speak of it as subsisting is merely to attempt to smuggle it back into reality, from which it has been once dismissed.

An expression which "means a non-entity" stands, then, for no sort of entity; but it is still an expression which has significance and an objective reference, apart from the existence or subsistence of any corresponding referent.

V

Under what conditions is reality or existence predicated of objects? What are the criteria of objectivity?

The first criterion of existence, as the term is used in the present definition of truth, is perception. In perception, something is given to the mind, there is a *datum;* and it is to this datum that existence belongs. And yet an uncriticized perception does not guarantee the reality of its object. Dreams, illusions, hallucinations, are no less vivid and convincing than waking or normal presentations; they are no less perceptions in which something is given. But one cannot say that a dream object or an illusory object is really what it appears to be.

Perception is complex; the datum is only one element in it. What seems to be given as a datum may be largely the creation of concepts, of signs with their attendant mental attitudes, at work in the perception. For an object perceived is always an object meant, referred to, intended by, a concept. Concept and datum blend to form a presented whole, and what is merely intended or meant cannot easily be separated from what is given. Perceptions must be criticized, and only some of them can be accepted as presenting objects which are *really* what they seem to be.

I am walking in a forest, let us say, and I see at some distance the body of a man across my path, which proves on closer inspection to be the trunk of a fallen tree. What I first saw had the

appearance of being given; yet it could not have been wholly given or it would have resisted further examination. The perception was an elaboration of data. Images, and perhaps unspoken words, together with their meanings, fused with the data; the mind automatically organized these elements into a whole, so that the data were presented under a concept — they were perceived through a meaning. Not only was this the case with the perception of the recumbent body; there is every reason to suppose that the perception of the tree-trunk was also an elaboration of data. Here, too, concepts were added to what was given, the data were viewed through a meaning, the presentation being no less complex; and if I believe the fallen tree-trunk to be a real object, it cannot be on grounds of its givenness alone.

Perception is a union of sensation and conception. Sense data are the simplest observable elements in perception; with them the analysis of a presented whole comes to an end, though it is difficult (and often impossible) to disengage these sense data from their settings. The sense data are obviously supplemented by something non-sensory. They are signs which, in the language of Thomas Reid, "suggest" what is perceived. Take an extreme example: I hear a footstep outside my closed door and perceive that someone is about to enter the room. The sensory elements are plainly much less than the whole perception. The sound may not be a footstep, it merely "suggests" a footstep; and certainly it may not be the footstep of someone who is about to enter the room. This is clearly a conceptual addition to the sense data. Now what is true in the extreme case is true also in other cases. One perceives, for example, that it is raining; but the sensory constituents of this perception are not the whole fact, *that it is raining*. There are sensations of dampness, of grayness, of cold, of faint vertical motions in the field of vision; and these sensations together with what they mean make up the fact. They awake in the mind, as do the words on a printed

page, certain intentions carried by images and half-uttered words; they cause one to take an attitude which is appropriate not only to dampness, to cold, to grayness, but to a whole of which these are parts; and thus they mean that it is raining. The effect of them is as if they caused one to formulate this proposition.[1]

The sense data and the concepts, moreover, come into a peculiar unity in perception. They coalesce, and the elements that are sensed infect the conceptual elements, so that a large part of what is conceived seems also to be sensed. An excellent illustration of this is the perception of space; we seem actually to see depth, though there is no doubt that much, if not all, of the perception of depth is a conceptual elaboration of sensory elements. The sense data are both signs to which concepts attach and parts of the objects which these concepts signify; and in the perception of a complex object or fact the sensory elements fall into their proper places as constituents of a whole which is both conceived and sensed.[2]

The presented whole, nevertheless, has the appearance of simplicity. The fact *that it is raining* seems to be given in its en-

[1] Reid adds that in perception we believe "irresistibly" in the reality of what is presented, and so perception becomes judgment, being conception joined to belief. But perception need not include belief. One often doubts the evidence of the senses; yet the doubt does not do away with the perception. Further, an illusory perception persists when it is disbelieved; this is indeed the very reason why it is illusory. It is nearer to the fact to say that we tend to believe our perceptions, that the perception tends to become a judgment, rather than that it is a judgment. Belief is a new attitude of mind, and one can withold belief in his perceptions. If perception produces immediate and "irresistible" belief, this is a belief that usually gives way to doubt — a belief that does not persist unless it is substantiated by other evidence than the givenness of the perception in question. See T. Reid, *Essays on the Intellectual Powers of Man*, Essay II, especially ch. 10.

[2] A thing, a complex object, is therefore much more than a class of sense data, as Mr. Russell describes it. It is a unity of sense data according to a definite plan of structure. The data enter into a conceptual scheme, they become parts of a meaning, and are at once signs and constituents of the object meant. For Mr. Russell's view, see *Our Knowledge of the External World* (1914), ch. 4.

tirety; the tree-trunk appears to be presented as a solid, three-dimensional object; and even though the perception is known to be a fusion of data with concepts, an inspection of the presented object by itself does not readily separate what is conceptual from what is not conceptual. It may well be that the object exists as it is perceived; that the very whole intended is given, as well as the sensory elements. The concepts may aid the mind to grasp what is actually a complex datum, rather than cut it off from a clear knowledge of the real object. And yet if the whole content of every perception is taken to be a datum, it is impossible to deny existence to the objects given in illusions, hallucinations, and dreams. If I dream that I am in Thibet and awake to find myself in my bed, I have been in Thibet and have been miraculously transported to my bed. The conclusion must be that, in some perceptions at least, only a part of the content is a datum; in illusory perceptions, the interpretation of the data, rather than the data themselves, creates the illusion.

There is a compulsion in perception which is not found in conception, and this is why we conclude that a part, if not all, of the content of the perception is given to the mind and exists. No analysis of knowledge can ignore this compulsion of perceptions; this is their claim to be presentations of reality. Concepts can be altered at will so long as their references to perceptual objects are disregarded; they are inventions rather than data. But perceptions — even illusory and dream perceptions — are not purely invented. There may, to be sure, be some metaphysical sense in which the data of perception are created by mind; nothing may be completely independent of mind in the most inclusive meaning of the term. The data of perception may be nothing more than lively and compelling psychical states — nothing more than Hume's *impressions;* or they may be, as Berkeley would have us believe, thoughts in the Divine Mind.

But this dependence on mind in a wider sense would not render them, if they are truly data, any the less independent of thought in a narrower sense, that is, of the mind's conceptual or symbolic activity as it has been described. If conceptual activity created objects, there would be a real object for every thought; ontological proofs could not be invalid; one could not think without thinking of the existent, and even a phenomenal distinction between reality and illusion would disappear. Existing objects in the limited sense now in question are independent of concepts. Yet these objects are cognized in perceptions from which concepts are never absent.

The most that can be said of the reality of a datum, apart from the concepts through which it is known, is that it exists as "something-or-other"; and this is self-evident and trivial, for it asserts merely that it is a datum. Such an indefinite predication of existence is not yet the cognition of an object, and is not sufficient for organized knowledge. To apprehend the reality of an object is to apprehend more than the existence of "something-or-other," more than reality in general, though this wholly unspecified reality is the only reality that givenness alone can confirm. The "something" which is real must be perceived to be of a definite nature or in definite relations to other things; it must be *placed* in reality, as well as perceived simply to exist. It will then be brought under a concept; it will be apprehended through a meaning.

No presentation can be dismissed as totally unreal, however illusory its content may be. In a dream, the dream places, people, events, are given to the mind and they exist. But they do not exist as physical realities; they are dream objects only. The fact that I perceive palaces, gardens, and praying fakirs in my dream of Thibet is no illusion; the illusion is that I perceive these as physical rather than dream realities. The deception

vanishes when the data are differently conceptualized — when the whole is placed in reality as a dream.[1]

When reality is predicated of what is given in perception, this is always a predication of something other than bare reality; it is a predication of reality of a definitely intended sort. If, on the contrary, the content of a perception is judged to be unreal, this judgment does not mean that it is unreal without qualification. It is unreal only under some concept, *e.g.*, as a "physical" or "external" object, etc. Bare existence belongs to every datum, and the question for knowledge is: How can this existence be so conceptualized that it fits in with other existences?

If perception were the apprehension of data and nothing more, everything would be what it is perceived to be, and givenness in perception would be an absolute guarantee of the reality of the object given *as* it is given. But a presentation from which all conceptual elements were subtracted could be only an immediate awareness of a reality that could be named or characterized in no way. The datum would not be articulately experienced; it would be intuited rather than perceived. Truth, as a property of symbols that mean existent objects, is not a reference to reality as it might be known in pure immediacy, but a reference to perceived objects; and givenness in perception goes only part way toward assuring reality to such objects.

VI

Do not sensations guarantee the existence of their objects, if perceptions do not? Are not sense data "hard," indubitable, and free from conceptual elaboration?

[1] "Life and dreams are leaves of the same book. The systematic reading of this book is real life, but when the reading hours (that is, the day) are over, we often continue idly to turn over the leaves, and read a page here and there without method or connection; often one we have read before, sometimes one that is new to us, but always in the same book. Such an isolated page is indeed out of connection with the systematic study of the book, but it does not seem so very different when we remember that the whole continuous perusal begins

Sensationalism and empiricism have been frequently confused. It has been taken for granted that what is given in experience is given through the senses alone and that there are no data but sense data. And yet, search as we may for a pure sensation, one is never found. "A pure sensation is an abstraction."[1] Sensations appear only in the context of perception, as the simplest elements of presented wholes. Experience itself is of "a teeming multiplicity of objects and events," which cannot be reduced to or derived from elements so thin as sense data; and the attempt to do so is well characterized by James as "abandoning the empirical method of investigation." [2]

To recognize a sense datum is to observe that the complex objects of perception can be analyzed into simpler constituents, but this observation does not carry the implication that these complex objects are (or are known as) sense data and nothing else.[3] The table on which I write is brown, hard, and rectangular; it is cold and smooth to the touch; yet these sensations, by themselves, do not give the table I perceive. The perceived object is more than sensory elements; it is these "thought" or meant together into a whole. A pure sensation, or collection of pure sensations, if there could be any such thing, would be as near the negation of experience as anything imaginable.

Short of a purely intuitive knowledge, which would be an awareness of no specific object or quality — not even of something as definite as a sensory quality — there are no cognitions which do not make use of concepts. The elements in experience which psychologists call "sensations" only approximate toward pure sensations. The concepts through which the content of a

and ends just as abruptly, and may therefore be regarded as merely a larger single page." Schopenhauer, *The World as Will and Idea*, Bk. I, sec. 5.

[1] W. James, *The Principles of Psychology*, ii, 1.

[2] *Op. cit.*, vol. i, ch. 9, p. 224.

[3] The view which makes sense data the primary units of cognition speaks of objects, situations, facts, as "constructs" from sense data, but we are urging that sense data are "destructs" from complex objects, situations, or facts.

sensation is apprehended may have been thinned down to a minimum; yet it cannot be maintained that concepts are wholly absent so long as sense data are known as specific qualities, so long even as they are definitely attributed to the senses. The very predicate "sensation" conceptualizes them. The cognition of objects, qualities, events, no matter how simple, is never anything less than recognition; the mind intends, takes an attitude appropriate to, the things it senses, and thus recognizes them as things meant. A bare sensation, *e.g.*, of *red*, does not spring into my mind without a concept to meet it. My mind is prepared for knowing red, and the sense datum stands forth from the background of purely immediate knowledge because "red" is what I mean as well as what I sense. And if I mean "red," a sign of some sort is present. There is an image or an incipient vocal utterance, etc., which carries what is in effect the proposition, "this is red." Through the concept, the sense datum becomes a part of my articulated knowledge. Sensation then "differs from perception only in the extreme simplicity of its object or content"; [1] and the outer limit of knowledge is not sensation but the pure awareness, or intuition, which surrounds the whole act of concrete knowing.[2]

Since conception is present everywhere in knowledge within this outer limit, the richer experience of objects, events, situations, facts, has no less a claim to be taken as a datum, which exists as experienced, than have sensations. The true empiricism is not sensationalism. The true empiricism accepts the content of presentations in their wholeness and makes what it can of them, rather than analyzing out the sensory aspects and denying reality to all that remains. Sensationalism does violence to experience and is the shortest road to scepticism, for, if only the impermanent and insubstantial data of the senses exist, reality speedily dissolves into a flight of subjective shadows.

[1] W. James, *loc. cit.* [2] See below, ch. VIII, sec. xii.

The fact that experience in the full meaning of the term appears only when concepts, that is, thoughts and their references, enter in cognition, does not make it necessary to hold that these conceptual factors cut us off from reality. Concepts may facilitate as well as hinder the apprehension of data, for it is only by means of concepts that the passing sensation is arrested and experienced as an aspect of an object. When a landscape is before me, I know through the concepts that I am experiencing trees, mountains, and a river, rather than a mere flow of sensations. The concepts may cause me to come into a closer cognitive relation to the real, instead of causing me to misinterpret and misconceive the real. Thought is not a veil separating the mind from what exists.

Kant shattered Locke's conception of the mind as a *tabula rasa;* he made it evident that mind is active not passive in perception. Even Locke faltered in his "blank-sheet" sensationalism. The mind, which, like a dark room with a single window high in the wall, has at the beginning of Locke's *Essay* only one aperture of light, sensation, is discovered in the end to gain most of its "complex ideas" from comparing, relating, and abstracting the "ideas" which enter through the senses. The mind is found to be active. Knowledge will not submit to the *tabula rasa* description.

At every point in experience, thought — organization in a conceptual scheme — is at work. Concepts, symbols and their intentions, bridge the gaps between *this* experience and *that*, between disorganized sensory presentations and integrated perceptions; and empirical reality — reality in the sense in which we are now speaking of it — comes into knowledge through what might be called "presentational thinking."

The view that thought isolates the mind from reality and leaves a thing-in-itself shivering in the empty noumenal spaces beyond experience, might possibly be true for an ultimate real-

ity.[1] But Kant himself insists that the objects of experience, if they are "transcendentally ideal," are "empirically real." They are given in presentational thought, and this is their empirical reality. Whether or not reality is, in the last resort, inaccessible to thought, there must be some sense of the term "reality" which makes the real accessible to thought; otherwise there would be no objects of perception. Objects are capable of existing, in this meaning of the term, as they are perceived through concepts — as presented wholes of definite characters or in definite relations; and truth is reference to these objects.

Thought, therefore, may cause knowledge to approach the real, or it may remove it from the real. Truth in perception is possible for the same reason that illusion (or falsity) is possible; because thought is a constituent of perception. And for this reason, likewise, no perception in itself attests the existence of its object as it is perceived.

VII

The second criterion of existence, which is a necessary supplement to givenness in perception, is consistency — consistency with the whole of our organized knowledge. By means of this criterion, the data of any single perception are fitted in with other data and given a place in reality, as well as judged barely to exist. At the same time, this criterion permits us to infer with more or less certainty, depending on the strictness of the inference, the existence of objects not given in perception.[2] (That this criterion of existence — consistency with the whole of knowledge — does not render our definition of truth circular will be presently shown.)

[1] This metaphysical hypothesis, however, is extremely unstable, since the thing-in-itself can be positively characterized in no way. It is an x, of which one can say nothing more than that it does not appear in any experience; it is something other than any possible object of experience. See below, ch. VIII, sec. iii.

[2] The whole problem of the validity of induction is involved here. But if any inductions are true, the objects they refer to exist.

The axiom that there can be no inconsistency in the real is open to several interpretations, each of which turns on a different notion of reality.

Consistency and inconsistency are relations between propositions or concepts — that is, between symbols. By definition, p and q are inconsistent if, when one of them is true, the other is never true. No two true propositions are inconsistent. Now, on the assumption that an existent object is not a proposition or a concept, it follows that the real cannot be truly said to be eithor consistent or inconsistent, though the concepts through which it is viewed give it the appearance of consistency or inconsistency.

The thorough-going rationalist, if he grants that reality is not conceptual or propositional, will still believe that a complete knowledge of the real through propositions would be a consistent knowledge. He will postulate that reality is consistent in the sense that it can be consistently conceptualized, even though it is other than concepts. The anti-rationalist, on the contrary, will assert that no consistent or inconsistent body of propositions is adequate to reality; that the real is wholly beyond conceptual knowledge and is known no less truly through inconsistent propositions than through consistent ones — for, to the anti-rationalist, neither could represent reality. Thus the thesis of any one of the Kantian antinomies tells us as much, or as little, about the Kantian reality as the antithesis; and M. Bergson joyfully embraces contradictions as a means of leading the mind to intuitions, not because reality is contradictory but because it is beyond the grasp of concepts and propositions. "In attempting to describe what we know in the abstract logical terms which are the only means of intercommunication that human beings possess, Bergson is driven into perpetual self-contradiction, indeed, paradoxical though it may sound, unless he contradicted himself his description could not be true." [1]

[1] K. Stephen, *The Misuse of Mind* (1922), p. 12.

Each of these interpretations of the axiom of the consistency of the real — the rationalistic and the anti-rationalistic — refers to an unlimited, an ultimate, reality. But the reality we have been considering is limited, and the very principle of its limitation is that it must be capable of being consistently presented and represented. If there are realities which cannot be consistently conceptualized, these are real in some more extended sense. One cannot deny Kantianism or Bergsonianism unless it be on metaphysical grounds; nor can he affirm the complete consistency, the rationality, of the real. It may be that empirical-rational knowledge floats on the surface of a world whose depths are reached neither in perception nor in thought. Yet it is possible to go some distance with a less extended notion of the real, which makes reality rational but does not preclude a final "irrationalism."

We mean then by a real object one which can be given in (or inferred from) an experience which is consistent with the whole of knowledge. Such real objects can be known through concepts, as these concepts enter in perception; and if an object given in perception is really what it appears to be, its characters and relations will not have been created by the act of conception. A real object is independent of concepts, though it may not be independent of *mind* on a much wider interpretation of this term. Moreover, these real objects must have a structure; otherwise, they could not be known through concepts, nor could they be consistently assimilated to the whole of knowledge.

These are all necessary conditions of the reality of an object, in the present limited sense, but it must be observed that they are not sufficient conditions. The whole of knowledge might conceivably be false, and then none of the objects presented in such a consistent experience would be really what they are perceived to be. The most we could truly say of them would be that they had a bare existence, that they were not non-entities. And

so we have not completely defined even this limited reality; we have merely laid down certain conditions which it must fulfil, we have confined it within certain boundaries. It remains still a basic and undefined term, necessary to the analysis of truth.

VIII

For Kant the limits of experience were marked by the categories, which were necessary principles of thought, implicit in the act of judgment itself. But Kant's categories are empty of content. The consistency of empirical-rational knowledge is more than a purely formal consistency, more than fidelity to the laws of logic — even of "transcendental logic." Everything that can be thought or experienced will, it is true, obey these formal laws, but obedience to the laws of logic alone is no criterion of existence. The touchstones of the reality of what is presented to us are general truths which cannot be derived from the principles of logic. They are categories, but not categories which are implicit in the nature of thought; no *a priori* deduction of them is possible; they are always subject to revision.

First among these "empirical categories" can be placed the generalizations of common sense that is, prescientific generalizations: nothing is ever both crooked and straight, round and square, black and white; nothing is ever before something else and at the same time after this same thing; nothing is ever outside something and at the same time within it; everything has a cause; water does not flow up-hill — and countless other principles which do not follow from the postulates of logic but which operate in determining the reality of objects. When I awake from a dream of the other ends of the earth, though the experience has been sufficiently vivid to convince me of its physical reality, I conclude that I was dreaming; for I could not have been in two such distant places as the remote regions of the earth and my own bed at so nearly the same time. The dream is

inconsistent with common-sense physical categories. So long as I conceive these events as physically real I am unable to place them in experience without contradiction.

All well-established scientific principles operate in the same way as the generalizations of common sense in determining the reality of given objects. It is assumed that if an object is really spatial or numerical in its nature it will not violate the principles of geometry or arithmetic; that no material reality would transgress the laws of mechanics, and no chemical reality the laws of chemistry. Every new generalization, every new theory, adds a further category under which reality is predicated of presented objects; so that experience both tests and is tested by scientific and prescientific generalizations. Laws, theories, and the facts on which laws and theories are built mutually correct one another. A generalization is verified by the existence of an object which conforms to it in a special instance, or by the empirical reality of something which can be deduced from it. At the same time, presented objects are accepted as real because they are perceived as special instances or consequences of general principles. The criterion of the truth of theories is empirical reality, and the criterion of empirical reality, the truth of theories.[1]

We may be mistaken in our perceptions themselves, or in the assumptions, the body of theory, by which we estimate the reality of what is given in perception, or in both. It is always possible that a new set of categories may completely overturn all previous estimations of reality. Yet the fact that all perceptions and all judgments are subject to error does not make it necessary to believe that no perceptions, no judgments, give us reality. When a presentation is consistent with the whole body of knowledge, the presumption is overwhelmingly in favor of the presented object being really what it is perceived to be. We can

[1] "Theory" is here taken to mean any principle or body of principles that exceeds the given.

attain no more certainty than this. There is no self-evident, undeniable badge of reality — unless it be of bare existence. The categories that operate in fixing the places of presented objects in reality are not *a priori* and unalterable principles of reason, blank forms of perception and judgment. They are accepted universal truths which are themselves subject to error. The mediaeval philosopher who, when he was asked to view the spots on the sun through a telescope, replied — "I have read Aristotle many times and I assure you that there is nothing of the kind mentioned by him; be certain therefore that the spots which you have seen are in your eyes and not in the sun" — was correct in method though not in fact. His categories were antiquated, but within these categories his reality was consistent.

A nice balance of theory and fact is for science the very principle of its life. And if the "delicate, contentious, and fantastical" learning of the Middle Ages bore no fruit but the delight in speculation for its own sake, the worship of fact alone cannot bring forth even this fruit. Knowledge is a growing whole of fact and theory.

IX

If truth is the correspondence of concepts or symbolic expressions with existent objects, and an existent object is one given in a perception that is consistent with the whole of knowledge, is not the definition of truth completely circular?

It would be circular if consistency with the whole of knowledge were identical with existence, that is, if concepts created their own objects. But this is not the case. The existent is independent of the concepts through which it is known. Truth as a property of symbolic expressions, and existence as a property of objects, are not the same. Though existence can be definitely predicated of objects only if certain general truths are consistently applied in experience, the objects to which these truths

refer stubbornly resist being reduced to thought-creations. They are data given to thought but not invented by it, and truth is reference to reality, not reality itself.

Consistency with the whole of knowledge is therefore a test of existence and of truth, but it constitutes neither truth nor existence. Clearly, if truth is the correspondence of concepts with existing objects, no true generalization could be inconsistent with a perception of existing objects. True generalizations would necessarily become tests of existence. But the existence of objects would not thereby become a function of the truth of these generalizations; nor would the truth of these generalizations be merely their consistency with perceptions of existing objects. The tests of truth follow from the definition; they are not equivalent to the definition.

Moreover, the consistency which is a function of the truth of propositions [1] could not define truth, for *its* definition presupposes the idea of truth. And if consistency means, as it otherwise would, "conceivability" apart from truth or falsity, this is equally far from being truth. One can invent an infinity of consistent systems of concepts, if consistency is a matter of conceivability, of definition, alone. Such systems remain non-contradictory so long as the ideas in them are used with the original definitions, and so long as the general principles of the construction of concepts are observed. But there is no compulsion, no reason other than caprice, for defining an idea in one way rather than another. The consistency of such a system gives it no reference to reality; it places it merely as one among an infinity of "possibilities for thought." If I find perceptual objects that can be consistently conceptualized in the propositions of such a system, the system has passed one test of truth; and if I further discover that it is consistent with the whole of accepted knowl-

[1] p and q are consistent if there is a case in which both are true, and inconsistent if, when the one is true, the other is never true.

edge, it has passed another. Its truth is as nearly established as possible. But the proof of truth and truth itself are different things. The truth of the system is its correspondence to reality.

X

The pursuit of "conceivability" as a definition of truth leads to the "coherence" theory, in which conceivability finally transcends the limits of all ordinary thought. The essence of the correspondence theory is that it holds thought and reality apart, and the essence of the coherence theory, that it identifies them — that truth, reality, and conceivability become one. Coherence, as the term is used in this theory, is more than the consistency of a single science or of all sciences. It is an ideal of knowledge beside which all else is fragmentary and "mutilated," and to which no finite knowledge attains.

This is made plain by Mr. H. H. Joachim, who carefully distinguishes coherence from consistency: "The 'systematic coherence,' therefore, in which we are looking for the nature of truth, must not be confused with the 'consistency' of formal logic. A piece of thinking might be free from self-contradiction, might be 'consistent' and 'valid' as the formal logician understands those terms, and yet it might fail to exhibit that systematic coherence which is truth." [1] For Mr. Joachim, to be "conceivable" and "coherent" means to be "a significant whole"; and ultimately there is only one significant whole — the Ideal Experience, which is Reality. Nothing short of knowledge of this Whole, this Absolute, is Truth. There are no separable truths. A single isolated proposition is partly true and partly false; it is incomplete, and no more than a faltering step toward the perfection of knowledge — the one Truth. Mr. Russell, in criticising Mr. Joachim, pointedly asks : Is the theory itself, being not the whole of knowledge, wholly true? [2]

[1] H. H. Joachim, *The Nature of Truth* (1906), p. 76.
[2] B. Russell, *Philosophical Essays* (1910), p. 150.

Mr. Joachim's examination of truth proves one thing certainly: that truth, as it appears in empirical knowledge, in science and every-day thought, and in the despised reasoning of the formal logician, is not the truth he is speaking of; that a metaphysical criticism of knowledge passes beyond — and it may be reverses — a descriptive epistemology.

The objections that Mr. Joachim levels against correspondence are instructive; they illuminate the nature of this relation.[1] He points out that any correspondence must be a correspondence of the structure of wholes as well as of their elements. Each element must play the same part, fulfil the same functions, as the corresponding element in the other whole; and thus the two must be identical in structure. But he concludes that it is impossible to make out any such correspondence between perceived wholes and conceptual or propositional wholes, giving as his reason for this that: "On the one side, we have a whole of experience at the level of feeling; and, on the other side, a whole of experience at the level of reflective thought. To say that there is (or may be) identity of structure is to maintain that these experiences are different matters subsumed under an identical form . . . the idea of an identical structure in different materials is quite inadequate when applied to the wholes in question, *viz.*, felt- and thought-wholes." [2]

The conclusion which Mr. Joachim rejects is exactly the one that the analysis of logical form has led us to accept. Concepts, symbolic expressions, and presented objects are indeed "different matters subsumed under an identical form." A perceived

[1] There is no doubt, as Mr. Joachim shows, that unless reality — or a kind of reality — were present in knowledge, the notion of truth as correspondence to existent objects would be an unworkable hypothesis. Locke, for instance, seems to be beating thin air when he describes truth as the reference of ideas to unknowable objects beyond ideas. Yet even this theory cannot be shown to be false. The most serious accusation that can be brought against it is that it is quixotic; that it is impossible to apply the definition as a test of truth.

[2] *Op. cit.*, pp. 26, 29.

whole is not merely "felt"; perception is not on the level of pure awareness in which nothing articulate is given. Objects are perceived as having a certain structure, their elements are given as grouped in a definite order, they are composed of major and minor groups of distinct and recognizable forms. Reality, in the present sense, is rationally presented in that it is analyzable, in that its form is presented with it. And what Mr. Joachim calls the structure of "reflective thought" is embodied in the grouping of the symbols in which the thought is expressed. The identity of form in propositions (symbolic groups) and perceived objects brings about the correspondence of "reflective thought" with fact, which is truth.

Correspondence it is true implies a distinctness, a separateness or externality of the corresponding wholes. Yet in perception there is no separation of the concept, which is a constituent of the presentation, from the datum. Datum and concept cannot be disengaged and compared. Must one say that here the relation is no longer correspondence, that datum and concept become one in "presentational thought," where the margin of distinctness between them is so narrowed that the object is given along with (or through) the proposition which is true of it? This is undoubtedly Mr. Joachim's view; a perception, he believes, is already at the level of "reflective thought"; there is nothing external to the perception to which it can correspond.

A pure concept, that is, a proposition whose object is not given in perception along with it (or through it), is, on the other hand, clearly distinct from its object and can be said to correspond to the object; *e.g.*, "Napoleon escaped from Elba" is purely conceptual, its object does not accompany it, and hence there is no difficulty in describing its truth as correspondence to fact. Scientific theories, generalizations, historical statements, memories, predictions — the larger part of knowledge — are purely conceptual; such propositions are distinct from the ob-

jects of which they are true and can be said to correspond to these objects. But, as Mr. Joachim points out, this is an unimportant type of correspondence, and it cannot be truly said to be correspondence — but is rather coherence — unless the truths of perception can be likewise shown to be genuine cases of correspondence.

There is no doubt that in perception the proposition and the object cannot be torn apart, the concept fits the object as the die fits its cast, the two coincide as superimposed figures in geometry might; and the only evidence of a discrepancy is the inconsistency of the perception with other experiences. Yet the relation however close must still be correspondence when the concept is true, for so long as the concept and the object are *not identical* they are distinct. One would be justified in giving up the notion of correspondence only on the hypothesis that the proposition and the datum are the same. And this is the fundamental reason beneath the objections of the coherence theory to correspondence as a definition of truth: the coherence theory wishes to eliminate the distinction between thought, or conception, and reality. Certainly if an existent object is identical with a coherently thought object, it is absurd to speak of truth as the correspondence of concepts with existent objects. But existence must be independent of concepts, where conception is anything short of the coherent — and unattainable — thought which is, for Mr. Joachim, Reality; especially, where conception is the significant use of symbols. For if existence were identical with conception, every thought would refer to a real object. We could not escape thinking of the existent; there would be no falsity. Since the existent and the conceptual are distinct, their relation is correspondence, however narrow the margin of separation for knowledge between them; nor does identity of structure make the proposition and its object one.

The notion that the structure of thought is found in the sym-

bols it uses to express itself and that this same structure permeates the world of real objects, at least the real objects which can be presented in a consistent experience; the further notion that these real objects are not identical with concepts and yet can be apprehended only by the use of concepts, though they may in this way — even in perception — be apprehended falsely; these ideas give a complete and simple meaning to the definition of truth as the correspondence of concepts to reality. At the same time, the notion of syntactical significance, of meaning which is directed toward the world of objects yet need not terminate in a single object or fact, permits propositions to be false without robbing them of their significance. Truth becomes in a literal sense a property of symbols, for propositions and concepts are symbols or symbolic groups as they function in minds. All this is implicit in Hobbes's statement that "true and false are attributes of speech, not of things . . . truth is the right-ordering of names."

XI

Since meaning alone is all that is necessary to truth and falsity, belief and disbelief, assertion and denial — in short judgment — fall outside the discussion of truth and falsity proper. Yet from one point of view, judgment is closely connected with truth, for no truth is genuinely known until it is judged or believed. To know truth is to judge, but to be in doubt, to entertain ideas without assertion, to perceive without believing, is not to know truth.

Assertion and denial, belief and disbelief, are secondary attitudes toward propositions. They supervene on the primary attitude of understanding, which is that of holding the proposition, the symbol or group of symbols, before the mind with a consciousness of their reference but without regard to whether or not there is an object to which they refer. To understand is to frame a hypothesis, to put one's self in a state of inward prepa-

ration for an object when the mind is still in doubt as to the existence of such an object; and here the attitude of understanding ends, or else passes by a gradual transition into that of belief.

To believe an idea was, for Hume, to have it vividly present in consciousness; but the vividness of ideas is a cause of belief rather than belief itself. It is certainly true that a lively perception of an object or an especially clear understanding of a proposition tends to become a belief. Entertain an idea for a sufficiently long time and you will believe it; [1] for belief is of the same mental genus as understanding. Both are preparations for objects, both are intentions of the mind with a direction beyond the mind's present content. Belief differs only in being a more complete preparation, a more earnest intention. Belief is readiness to act as if the proposition which is understood meant an existent object. It is willingness to use the proposition as if it were true.

As against Hume's notion that belief is the vividness, insistence, or compulsion of ideas, it is only necessary to observe that one may suffer from a vivid illusion without believing that the illusory content is what it appears to be. Though one tends to believe anything he understands (or its negative, which is suggested by it), and particularly to believe what is given in perception, still there is a point of transition between entertaining an idea or being presented with a content, and believing in the truth of this idea or the existence of this content. This transition cannot be described as a passage to clear understanding or vivid presentation, for the understanding may be already clear, the presentation already vivid. It is a transition to a new state of mind, to a state of mind which has something in common with the apprehension of meanings or of presented contents, but is nevertheless distinct.

The existence or non-existence of the things our thoughts

[1] Or believe its negative, which is suggested by it.

mean, the truth or falsity of concepts, adapts these concepts to uses which would not otherwise be possible. A true proposition can be employed as a premise in an inference in a way in which a proposition merely entertained cannot be employed. The truth and falsity of ideas makes possible a calculable and successful commerce with the world of fact; so that the value of truth is, in the first place, practical. Yet if truth is good as a means toward other ends, it is also good in itself — as an end toward which a certain intellectual interest is directed.

But there is an intellectual interest which is wider than the interest in truth: this is the interest in possibilities, in fancies or fictions for their own sake. The apprehension of significance, that is, of possibilities for thought (constructive imagination) is the primary function of the intellect, and if the truth of these possibilities added to them no qualities of use or attraction, thought would end in hypothesis. There would be no judgment; the existence or non-existence of objects meant would be of no moment. To the dreamer, the truth of his dream is irrelevant; to the believer, the truth of his belief is indispensable, for the motive of belief is the interest in truth. Thus beyond judgment lies pure speculation; add the interest in truth, which is the widest — the most impractical — of practical interests, and you add belief.

XII

Belief is more than the thought of existence coupled to a concept, and this is another reason why it must be regarded as a new attitude of mind, distinct from (though continuous with) conceiving and perceiving, however vivid and clear. To assert and believe that "Shakespeare wrote *Hamlet*" is not to entertain the thought of "the existence of Shakespeare's authorship of *Hamlet*," for this is itself a concept and not a belief. Belief and assertion take a step that carries the mind beyond conception;

they leap the gap between conception and existence by transforming the hypothetical into the categorical.

This is strikingly put by Hume in a passage in which he also shows that a single concept (that is, on our view, a single word or symbol) can be true or false and can be affirmed or denied. He comments on the accepted definitions of *conception*, *judgment*, and *reasoning* as follows: "Conception is defined to be the simple survey of one or more ideas: judgment to be the separating or uniting of different ideas: reason to be the separating or uniting of different ideas by the interposition of others, which show the relation they bear to each other. But these distinctions are faulty in very considerable articles. For, *first*, it is far from being true, that, in every judgment which we form, we unite two different ideas; since in that proposition, *God is*, or indeed any other which regards existence, the idea of existence is no distinct idea, which we unite with that of the object, and which is capable of forming a compound idea by the union. *Secondly*, as we can thus form a proposition which contains only one idea, so we can exert our reason without employing more than two ideas . . . . Whether we consider a single object or several; whether we dwell on these objects, or run from them to others; and in whatever form or order we survey them, the act of the mind exceeds not a simple conception; and the only remarkable difference which occurs on this occasion is when we join belief to conception, and are persuaded of the truth of what we conceive." [1]

As for Hume's first point, judgment is undoubtedly more than a union of the idea of existence with a concept. The ontological proof of God fails to produce belief because it is apparent that the addition of the idea of existence to that of God does not make the latter any the less a concept. But Hume, who is confirmed by Kant on this point, is not wholly correct in saying that "the idea of existence is no distinct idea, which we unite with

[1] D. Hume, *A Treatise of Human Nature*, vol. i, part iii, sec. 7, note.

that of an object." It is possible to conceive of the existence, or for that matter of the non-existence, of something meant; *e.g.*, "the non-existence of the round-square" is an intelligible expression. *Existence* can be a predicate, even if it is the most general possible predicate. The important fact is that, although *existence* is a predicate which can be added to concepts, belief goes further than the addition of predicates to concepts. It completely removes the concept from the area of the hypothetical.

The second point made by Hume in this passage is of special interest in connection with the present definition of truth. Hume declares that a single idea or concept can be judged and can be true or false, though truth is usually thought to belong only to propositions and it is not customary to give the name "proposition" to anything less than what is expressed in a complete sentence. "God," "evil," "the king of England," and similar fragmentary expressions are not commonly said to exhibit truth or falsity. But the difference between a word or a phrase and a full sentence is not that the one is capable of being true or false, while the other is not; the difference is that the one (usually) embodies an assertion or a denial, while the other embodies something merely understood, assumed, held before the mind without assertion or denial. "God is," "evil is," are assertions of what is meant by "God" and by "evil." The single words (unasserted) are therefore as much entitled to be taken as propositions (unasserted) as are any completed sentences.[1]

[1] Whether we are to name single words and incomplete phrases "propositions," is a verbal question. The term "proposition," if one so wishes, can be reserved for complex expressions which have the completeness of full sentences. But this will not alter the fact that the less complex expressions are either true or false. Propositions — even if they are held always to be complex, a union of subject, predicate, and copula — are unities which are affirmed and denied as wholes, and which are true and false as wholes. The subject-predicate logic, which asserts that the predicate is affirmed or denied of the subject, that it is true or false of the subject, breaks up the unity of the proposition and is at a loss to restore it. Yet this unity must be restored if there is to be truth. A proposition composed of a subject, predicate, and copula is true only if both

For some symbols, then, — for those which refer directly to objects and are not merely defined through symbolic groups, — truth and significance are the same thing. Knowledge, being built on elementary simple symbols which refer thus to objects, takes its rise from true concepts or "true symbols," and falsity, which appears later, rests on truth. Only when a kind of significance which is other than a direct reference to objects is devised does falsity become possible. But any symbol, either simple or complex, is either true or false, and a full sentence differs from a fragmentary phrase or a single word only in that the former is asserted or judged, while the latter is (usually) simply considered or understood.

XIII

Just as the primary attitude of understanding is carried by symbols, so also is the secondary attitude of belief. Beliefs do not hang in mid-air cut off from expression. One believes when he expresses this attitude by some sign to himself or to others, and since belief is readiness to act, a natural and common sign of belief is action. I know that I believe most of my perceptions because I act as if what appears in them were really what it seems to be; moreover, I am often unaware of what I believe until I assert it, which shows that belief is inseparably bound to its expressions.

the subject and predicate mean something existent, and — what is more important still — only if there is an existent whole in which the predicate is united by the relation of predication to the subject. The fact that the proposition can be broken up into a subject, predicate, and copula is not therefore the reason for its being true or false. This necessary unity of meaning in a proposition suggests that the term "proposition" can be used for any whole which has a unified meaning, and this would not exclude single words or incomplete phrases. We might speak of a single word as an "unanalyzed proposition" in contrast to all symbolic groups, which would be "analyzed propositions." Among analyzed propositions we should need to distinguish descriptive phrases as a special class. On this more extended interpretation of the term, a proposition would be any unified meaning; and since on our view meanings are carried only by symbols, it would be any symbol or group of symbols taken in its function as an instrument of significance.

Assertion is so closely related to belief that it is impossible sharply to distinguish them. If there is any difference it is this: that assertion is the inward or outward expression of belief by a sign, while belief itself is the attitude expressed. A full sentence always contains a sign of belief: "money is the root of all evil" stands for two things, for the fact and the belief in the proposition; while "money's being the root of all evil" signifies the same fact without the belief. (The passage from consideration to belief is often marked in thought by the insertion of the verb "is," or its equivalent, in the train of inward speech which passes and repasses in the mind.) If one openly asserts something he does not believe, the presence of the sign of assertion is nevertheless calculated to produce belief in others.

The copula or any symbolic element equivalent to the copula has a triple rôle in judgment. Its primary function is to call attention to the truth of the proposition in which it appears, to signify that this proposition is believed or ought to be believed. Secondly, it indicates the unity of the whole, though it is not necessary that the copula be present in order that the proposition be taken as a whole; the syntactical signs give the symbolic group its unity of meaning. Thirdly, the copula adds the predicate *existence* to the concept. But of these three functions, the first — as a sign of assertion — is the essential one.[1]

XIV

Belief and disbelief appear, on first thought, to be direct opposites which are related to one another as desire is to aversion, approach to withdrawal, or acceptance to rejection. Yet disbelief cannot be the witholding of belief, for this is understanding or consideration without belief or disbelief, without affirmation or denial. Nor can it be the dismissal of the propo-

[1] There is still a fourth meaning of the copula: its use to signify identity, *e.g.*, "Scott *is* the author of *Waverley*."

sition disbelieved as meaningless, for an idea must be entertained if it is disbelieved. Disbelief if more than the suspension of judgment, the refusal to commit oneself. It is as affirmative as belief.

"The true opposites of belief, psychologically considered, are," in the opinion of William James, "doubt and inquiry, not disbelief"; [1] and this suggests that disbelief is really a species of belief. Disbelief is a belief in the falsity of a proposition, that is, that there is no existing object corresponding to it; and thus disbelief is a belief in the truth of the proposition "*p* is false." By the principle of the excluded middle, either *p* or *not-p* is true, so that "*p* is false" is equivalent to "*not-p* is true," and "*p* is true" is equivalent to "*not-p* is false." Hence to disbelieve a proposition, to believe that "*p* is false," is in effect to believe its negative. (And the converse is also true; to believe a proposition is in effect to disbelieve its negative.) Disbelief, instead of being the negation of a belief, is the belief of a negation.

But if belief is an attitude of preparation, completer preparation than understanding, for an object intended by a proposition, how can one believe a negative? Are there negative objects? Can one act as if something negative were in existence? For this is what belief in a negative would require. If I assert that "I have not been in China" and expect this proposition to be believed, the assertion must refer to some objective content to which the belief can be fastened. Negatives must have a point of attachment to existing objects. This being the case, denials will take a place beside assertions as positive judgments which are equally useful in inference and equally necessary in bringing about successful adjustments to the world of fact. Full-blown disbelief differs from belief only in the nature of the content toward which it is directed; it is not a different attitude of mind.

Every proposition tends to suggest its negative. To under-

[1] W. James, *Psychology*, vol. ii, ch. 21, p. 284.

stand the meaning of a proposition is also to understand the meaning of its negative, and hence the intention which carries the significance may as readily pass over into disbelief as belief. To entertain the idea of *p* is also to entertain the idea of *not-p:* it is to be in readiness for two mutually exclusive contingencies. The added psychical pressure of belief may be in the direction of one or the other, but in either case this new attitude of mind will be continuous with, not opposed to, the attitude in which the idea is merely understood or considered as a possibility. When it is said that any idea which is understood tends naturally to be believed, this means that either its positive or negative tends to be believed; in other words, that the tendency to belief is always present in understanding but forks toward the two alternatives, one of which is disbelief.

There is, on the other hand, a frame of mind in which one experiences an intellectual aversion toward accepting either the truth or falsity of ideas, in which he says "no" without affirming or denying the proposition thus met; an attitude of simple incredulity, appropriate to strange phenomena or extraordinary concepts. It is this attitude which is directly opposed to belief. I may, like Aladdin before the hidden jewels, rub my eyes in the very presence of an unfamiliar sight, and be unwilling to act either as if it were real or unreal. This attitude, which is familiar to all men, completely replaces belief (and disbelief) in the mind of the sceptic, who knows no truth and is not even sure that he knows no truth.

Incredulity supervenes on and is provoked by the survey of possibilities for thought, but like its opposite, belief, it is a departure from the attitude of merely considering possibilities. The sceptic does more than understand or entertain the ideas of which he is sceptical; he sets his mind against them, he inhibits the natural tendency to affirm or deny the propositions he holds before his understanding. Incredulity is a definite shrinking

from belief or disbelief, and not simply the survey of possibilities for thought; and this is why it is the genuine opposite of belief. Thus in a mood of pure speculation or fancy, scepticism has no more place than belief or disbelief. It is quite as inappropriate to be incredulous of a myth or a romantic tale as to inquire whether it is true or false; to be sceptical as to whether the Ancient Mariner shot the albatross is no less irrelevant than to believe or disbelieve it. Yet to the thoroughly incredulous person, the true sceptic, a world of fancy — whether it be of romantic, scientific, or metaphysical fancy — is still open. Thought can range where it will among possibilities, for if possibilities for thought were not left to the sceptic, there would be nothing to be incredulous of.[1]

Scepticism is a sophisticated philosophy because only by dint of much thinking, much turning over of possibilities, do ideas so completely nullify one another that none seems worthy of acceptance. The unsophisticated man on the contrary is credulous, for concepts, ideas merely entertained, project themselves toward action and belief. Incredulity, when it is not erected into a philosophy, is unstable; it can play no positive part in knowledge. It can prevent us from falling into error and nothing more, but it can be of no use in shaping a course of action, in arriving at conclusions either in theory or practice, as positive denial can be. The "no" of the sceptic is the "no" of indecision. To be incredulous is to attain no end, but positively to deny a proposition is to believe something which may be of use.

XV

Belief and disbelief, denial and assertion, are the only avenues of approach to truth. Though truth is not manufactured by

[1] Aristotle denies even the consideration of possibilities to the sceptic, for he believes that one must accept the truth of the principle of contradiction if he *thinks* in any sense of the term. See Aristotle, *Metaphysics*, bk. iii, ch. 4.

judgments, it does not stand out for knowledge in isolation from the judgments through which it is known. The cognition of truth, like that of beauty or good, is colored by an interest, which is the motive of judgment — the interest in truth. And just as it is possible to lose sight of the objects toward which the moral and aesthetic interests are directed, and to confuse beauty and good with the feelings they arouse, so it is possible to mistake truth for the feeling of certainty that accompanies satisfactory belief. Truth is then reduced to a psychological state; it becomes a matter of intellectual taste, as beauty and good, on the parallel theory, are matters of aesthetic and moral taste.

The contradiction in this psychological relativism appears in the answer to the question which Plato put to the Sophists: If belief is the measure of truth, and I believe that this is not so, do I not believe truly? [1] There is no reply but an affirmative one on the Sophists' own hypothesis, and only if the relativist is willing to give up the law of contradiction, does his view sustain itself.

Scepticism, like relativism, often arises from the fact that truth can be known only through belief; but this philosophy, extending as it does no further than incredulity — incredulity reiterated and reinforced by incredulity — is born of too great rather than too little love of truth. The sceptic disdains to find truth in the feeling of certainty. He withholds judgment to the very last and does not deny truth, for he is aware that a single judgment — even the judgment that there is no truth, or that he doubts that there is truth — is like a stone cast in still water: the circles widen outward, embracing a more and more extended region, so that if one affirmation or denial is ventured on there is no stopping short of a theory of truth and reality. He therefore finds all things, perceived and unperceived, to be unacceptable to the intellect, indifferent to knowledge.

Hume's scepticism goes only part way. He is incredulous of all

[1] Plato, *Theaetetus*, Jowett's translation, marginal p. 161.

metaphysical theories, but not of the psychological processes by which ideas are joined together and brought into relation to the "impressions" of the senses. Yet he is acutely conscious that knowledge is belief — and, it may be, nothing but belief. If we could attain truth, he argues, we could not be certain that we had, for we should still be believing. The mind, like a moth attracted by a light, trusts its impressions because it cannot help doing so; but that these impressions are true of a reality given in them or beyond them remains forever no more worthy of belief than disbelief. Hume sees knowledge only from the psychological angle, and leaves it hopelessly tossed between *this* "irresistible" idea or impression and *that*, but with no anchor for any belief. And Hume, too, touches the profounder scepticism that doubts even itself; there are revulsions from philosophy, sudden returns to common sense, when the most ordinary beliefs seem to him no better or worse than the finest-spun philosophy.

Scepticism of this sort can be escaped but not refuted. One can easily imagine Hume undermining the whole structure of Kant's "refutation" by merely pointing to the fact that it is built on belief. Scepticism, in its subtler forms, is an over-cautious, not a self-contradictory, philosophy. It refuses to take the leap of belief — to undergo the risk of error that is never absent from knowledge.

The one final criterion of truth is our capacity to believe propositions; though a belief may be arrived at by circuitous routes, the ultimate evidence of truth is the compulsion of a belief. If one asks for what reasons he holds a conviction, it is either on the strength of some other conviction, or on the inherent strength of the conviction itself. Try as he may, he cannot break the circle of his beliefs. To test truth we must assume truth; we must believe that there is some valid principle by which truth can be tested, and that there is a truth to test.

It is not difficult for the sceptic to show the risk of defining or

attempting to test truth. Consider the test that has been proposed, consistency with the whole of knowledge. How complete must this knowledge be? Clearly, no experience is ever complete; there are always additions and corrections; in the light of further experience what now appears to be consistent may prove to be inconsistent. Yet if the test of truth is consistency with an incomplete experience, this is too narrow; no allowance is made for the reversals to which any growing body of knowledge is subject. But what could be meant by a "complete experience"? Is this something more than a personal knowledge—is it all knowledge, past, future, possible, including that of other minds, as it might be brought together in a single mind? If so, much as one might like to appeal to this complete experience, he cannot do so; for he can see through no other eyes than his own, and know through no other mind than his own; and he can know only in the present. The test he actually can apply is that of consistency in his experience at the present moment — the present including memories of the past; and this is only a small measure, a feeble approximation, to complete consistency. Furthermore, it may be that reality is not self-consistent, that consistency is read into it by the mind, that reality slips between the eager fingers of the intellect, leaving only a shell of empty appearances.

But the very imperfections of this test show how the circles of belief strive to push outward beyond the phenomenal to that which can in no sense be called an "appearance." The concept of truth links itself inevitably to that of reality. A truth grounded in anything less than a complete, a metaphysically conceived, reality will still be open to question in the mind that conceives this truth. A theory of knowledge without a metaphysics is at best only a partial repudiation of scepticism. If one embarks on an examination of truth, the security of his position will ultimately depend, not only on a faithful description of knowledge as a phenomenon, but on the scope of the concept of reality to

which he finally comes. He will need to complete the picture by a metaphysics of knowledge.

XVI

What has been said of truth, existence, and judgment can be summarized as follows:

The correspondence of propositions or concepts with existing objects is truth, but a proposition (or concept) is not a subsisting *tertium quid* interposed between symbols and objects; it is the symbols themselves, taken with the intention or psychical set they arouse in a mind. This does not deprive the proposition (or symbolic expression) of an objective reference, a direction toward the world of objects, even though there exists or subsists no referent for it as a whole. For its meaning is a function of simpler meanings united in a scheme of logical structure; and these simpler meanings, being direct references to objects that exist or have existed (or being definable in terms of such direct references), give the complex meaning a direction beyond the mind's present content. Moreover, the structure of the expression coöperates in determining the whole to have an objective reference, since the object meant (if there is an object meant by the whole) must be identical in this respect with the expression; and if the whole stands for no object, the structure, being a universal which could occur in a factual as well as a symbolic instance, will still refer beyond itself. The assumption of subsistent entities as the termini of meanings is therefore unnecessary; especially since meaning is not a static relation between the mind and some second term, but an activity — an activity of preparation or anticipation, which presupposes the presence in no sense of the object anticipated or prepared for. It is necessary only that some objects — those which are directly referred to by the constituents of the symbolic group — *shall have been* presented, in order that complex expressions should have a mean-

ing; these expressions in their entirety do not need to refer to any entity, subsistent or existent. Thus a false proposition is one which is significant, yet stands as a whole for no existing object or subsisting objective. The category of objectivity is no wider than that of existence.

The term "existence" or "reality" is used in a limited sense, though it is not defined; and the criterion of existence, which is also the criterion of truth, is the presentation of an object in an experience consistent with the whole of knowledge. Inferences based on such perceptions (if these inferences are valid) permit us to reach objects not given in perception. Every perception has a datum, but the datum is always perceived under a concept. Concept and datum coalesce to form a whole of presentation, so that what is given and exists independent of the perception cannot be disentangled from what might be added through concepts, by the mind's invention. Sense data are the simplest elements of presented wholes, but even these data are not purely immediate; the mind in recognizing them brings them under concepts. Therefore sensationalism, which grants existence only to the data of the senses and fails to observe that a sense datum is never known in complete isolation from presented wholes, is not the true empiricism; the complex perceptions of objects, situations, and facts have an equal claim to be taken as presentations of reality. Indeed, the concepts in perception may bring us into a closer cognitive relation to the real, rather than separate us from the real, for the mind is more than a blank sheet on which objects write impressions.

No presentation in itself guarantees to its datum anything other than bare existence, reality of an indefinite sort; but it does guarantee this thin reality even to the data of dreams, illusions, and hallucinations. When the data are apprehended as objects of this or that sort, or in this or that relation (as they must be apprehended if they are to enter into organized knowl-

edge), a further criterion of existence than *givenness* is needed. The data must be placed in reality under a concept which makes the presentation consistent with the whole of knowledge.

This consistency with the whole of knowledge is a criterion of existence for this reason, that the existence which gives truth, in the present meaning of the term, to symbolic expressions is restricted to objects which can be consistently conceptualized, which can be fitted into rational knowledge without contradiction. This does not assume that reality is finally self-consistent, or that it can be completely known through consistent concepts. Nor does it assume the contrary, that neither consistent nor inconsistent concepts are adequate to the real. That the existent is capable of being consistently presented and represented is the limiting condition of the notion of reality on which this definition of truth rests, but it need not be a condition of a more extended reality.

Consistency is determined not by the laws of logic alone, or by formal "categories," but by general truths, empirical categories, which are subject to revision and reversal. All well-established scientific principles and a host of prescientific generalizations operate as tests of the reality of perceived objects. The criterion of the existence of objects is the truth of theories, and the criterion of the truth of theories is the existence of objects. Theory and perception mutually support and correct one another. This fact does not make the definition of truth circular. Truth is correspondence to reality, and so it follows that perceptions of existing objects will be consistent with general truths, but it does not follow that existence is a function of these truths, nor that truth and consistency are the same. Consistency is a test of truth but is not equivalent to truth.

The "coherence theory" rejects correspondence as a definition of truth because this relation implies the distinctness of the corresponding wholes and hence the separateness of knowledge

and reality. The "systematic coherence" which is, on this view, at once Truth and Reality transcends the consistency of formal logic and ordinary thought. Thus there can be no separable truths, for such truths would not be the whole Truth. Though reality as thus conceived may be identical with thought as thus conceived, the limited reality of perceptual objects cannot be identified with the thought which is embodied in propositions or symbolic expressions, for if this were so there would be no falsity; every concept would refer to a real object. Since concepts and objects, in this less extended sense, are not identical but are distinct, the relation of concepts to objects, even when they come together in perception so that the meaning of the concept cannot be distinguished from the object, must be correspondence. This correspondence is a correlation such that the elements of the corresponding wholes fulfil the same structural functions in each; the relation of correspondence rests on an identity of logical form in real objects and in propositions or concepts.

Truths are known only through belief or judgment, which is a secondary attitude toward propositions. This attitude is like understanding in that it is a preparation for an object intended, but it differs from understanding in being a completer preparation — a willingness to act as if the object intended were in existence. Belief is more than the thought of the existence of the object meant, and more than vividness of presentation or clearness of understanding. Belief, like meaning, is carried in symbols, it is inseparable from its expressions; and if there is any difference between belief and assertion, it is that the latter is an outward or inward signification of belief, while the belief itself is the attitude of mind signified. A full sentence differs from a single word or phrase in that it contains a sign of belief; the essential office of the copula is to serve as such a sign. But a word or phrase is no less capable of being true or false, and of being

asserted or denied, than a full sentence; a single word is a proposition.

Disbelief reduces to belief, to a belief in the negative of the proposition disbelieved, while the true opposite of belief is incredulity, the attitude of the sceptic. Disbelief is therefore positive and takes a place beside belief as a means of reaching conclusions through inference or through action; incredulity serves only the negative end of preventing error.

Finally, the indecision of scepticism, which springs from the fact that truth is reached in no other way than through belief, can be escaped only by taking the risk of error which judgment necessarily involves; and belief, once it is set in motion, is carried on by its own momentum toward a theory of truth and reality — and in the end to "first philosophy." For nowhere else are to be found the principles that justify all others.

CHAPTER VI

NEGATION AND CONTRADICTION

I

THE notion of a negative truth — or a true negative — has an intrinsic appearance of paradox and even of self-contradiction, especially when truth is defined as the correspondence of propositions, or symbolic expressions, with objects.

What sort of object can a true negative mean? Does it stand for a "negative object"? Clearly it must stand for some object. The significance of a negative cannot be, like that of a false proposition, a meaning determined by other meanings and yet corresponding to no object, for all negatives would then be false. Nor can it be a direct reference to objects given in perception. There are no negative data; perception is always the perception of something that *is*, never of something that *is not*. A direct reference to objects cannot be anything but positive. The analysis of negation, therefore, falls between the horns of a dilemma: either negatives refer to no objects and are false, or they refer to objects and are not negatives.

There are many ways of escaping this dilemma, the simplest of which is to embrace one of its alternatives; and this is what Mr. Bertrand Russell does. He finds that negative propositions mean "negative facts," which are unanalyzed and necessary elements in the world of fact.[1]

A negative fact, if there is such a thing, is plainly not of the same order as a positive fact. To observe that "the sun is *not* shining" is very different from observing the contrary. In the

[1] See B. Russell, "The Philosophy of Logical Atomism," in *The Monist*, vol. xxviii (1918), and vol. xxix (1919).

latter case, a complex object, *the shining of the sun*, is immediately given; in the former, what is immediately given is something other than what is referred to. To "perceive" that the sun is not shining is to notice clouds in the sky and, perhaps, rain; or it may be, to observe that night has fallen — and these data are not negative facts. If negative facts are "given," they are not given through perception, as are positive ones; they are inferred or constructed from perceptions; their reality, like that of *classes*, extends far beyond any immediate presentation; they are facts of a highly complex type, and the propriety of using the term "fact" for them is questionable, as it is for classes, which cannot properly be called "facts." [1]

The assumption of negative facts as the referents of true negative propositions does not analyze negation. It cuts the knot but leaves the threads of the difficulty tangled. Negative significance remains an anomaly unassimilated to the rest of knowledge, unless it can be shown to have a positive basis in some other sort of meaning.

A particularly interesting attack on the problem is that of Mr. Raphael Demos, who, though he does not carry the analysis to its completion, suggests that a negative proposition is like a descriptive phrase; it is an ambiguous reference to what is meant by a number of positive propositions, the meaning of some one of which is the meaning of the negation.[2] Thus the sign of negation, "not" or its equivalent, plays a part similar to that of the signs "a," "any," "some," and "the." It is a sign of interpretation. It does not signify a constituent of the fact to which the expression as a whole might refer, but indicates that the proposition is to be interpreted in a special way, that is, negatively;

[1] See above, ch. IV, sec. xiii.

[2] See R. Demos, "A Discussion of a Certain Type of Negative Proposition," *Mind*, N. S., vol. xxvi (1917), p. 188. Despite obvious disagreements, the central idea of the theory of negation here presented, that of the ambiguity of the negative, is taken from Dr. Demos' article.

which is to interpret it as a variable with an ambiguous reference to certain positive propositions.

II

The ambiguity of negative propositions is their most striking characteristic. They are variables, and they continue as negations only so long as their significance is not fully determined.

If I assert that "the sun is not shining," I do not assert that "it is raining," or that "it is night," or "that the sky is clouded." None of these propositions expresses the meaning of the negation. Nor do I assert all of these propositions. The meaning is like that of the statement, "I met a man." If a value for the variable *a man* is chosen, so that the statement becomes, "I met Mr. *X*," this is not what I assert; it is more specific than my actual assertion. And though I must have met Mr. *X* or Mr. *Y* or Mr. *Z*, etc., if the statement is true, still it remains undetermined which one of these men I did meet, and this indeterminateness is an essential element in the meaning. Similarly, what I mean by "the sun is not shining" might be either that "it is raining," or that "it is night," or that "the sun is in eclipse," etc., but it is actually none of these. The negation is essentially unspecific in reference and to make it specific is to rob it of its negativity.

The observation that negatives are ambiguous is the first step toward freeing us from the error that a negative is merely *what is meant*. The *manner of significance* is no less constitutive of the meaning of a negation than are the things it might signify; and since propositions alone are ambiguous, negation belongs to the realm of propositions or concepts, and not to things. Things do not have negatives, for *things* are not ambiguous; and without ambiguity there is no negation.

Toward what objects or facts is this ambiguous reference of the negative directed?

Every negation has its ground; it rests on a perception or affirmation of fact. But the ground of a negation is not coextensive with the meaning, since the ground is a single fact or complex of facts, while the meaning covers a multitude of facts, some undetermined one of which is the value of the negative. I may affirm that "Chaucer's *Canterbury Tales* are not good reading" on the ground that they are immoral, but this fact will not be the only possible interpretation of the statement. This negation might mean that the language of these tales is difficult, or that they are dull. If I advise you that "carbolic acid should not be drunk," the ground of my advice may be that it is poisonous; yet this is not what I assert, and you may construe my statement to mean that "suicide is criminal" or that "life is worth living." To identify the ground of a negation with the meaning is to abolish the negation, for it is to choose one among the possible values; hence the negation is no longer a variable and no longer a negation. If a negative proposition were equivalent in meaning to the single positive proposition that is its ground, there would be no reason for employing negatives; they would disappear. Though negatives are often used where positive propositions would serve as well, their function in knowledge is the same as that of other ambiguous expressions; namely, to avoid a definite reference or to refer to that which is not determinately known. Negation is notoriously the language of diplomacy and guile.

In addition to the actual ground of any negation, there are many possible grounds. These possible grounds are all the propositions with which the proposition negated is incompatible; [1] and Mr. Demos limits the possible values of a negative to its possible grounds. On his view, then, a negative proposition means (ambiguously) the same as some one of the propositions

[1] Incompatibility is the relation of inconsistency between propositions, discussed in chapter v. p and q are incompatible if, when the one is true, the other is never true. This does not preclude both being false.

whose truth excludes the truth of the proposition negated. To say that "life is *not* short, nasty, mean, and brutish" is to refer to the fact that "it is long or happy," or that "it is romantic," or that "it is capable of perfection," etc., none of these possibilities, however, being chosen as the value of the negative.

Mr. Demos's analysis does not go far enough. The relation of incompatibility or logical opposition, in terms of which it is stated, is complex; it rests on truth and falsity and is an extrinsic rather than an intrinsic relation between concepts. It cannot be apprehended through direct inspection of propositions or concepts; one must know more than the meaning of a proposition to know that it is incompatible with another; he must know that it is never true when the other is true. Nor can incompatibility be apprehended as a relation between the data of perception. Logical opposition is not *perceived*, for what is meant by the logical opposites cannot *be* in the same universe; and if perceptions seem to be inconsistent with one another, the inconsistency is in the concepts through which the data are presented, and not in the data themselves. The proposition "*a* is black" is incompatible with the proposition "*a* is white," "Juliet loved Romeo" is incompatible with "Juliet hated Romeo"; but white is not logically opposed to black, and love is not logically opposed to hate, since all of these qualities or relations exist in the same universe. Propositions alone can be logically opposed, and their logical opposition is a function of their truth and falsity.

From the point of view of formal logic, there could be no objection to defining "incompatibility" in terms of truth and falsity and to deriving negation from this propositional relation. Nor could there be any objection to assuming incompatibility itself, without definition, as the foundation of a theory of negation and truth. It is also possible to construct a logical system in which negation and incompatibility are both defined in terms of another function of propositions. Mr. H. M. Sheffer's *rejec-*

tion is a primitive idea which makes these latter definitions possible.[1]

But what is primitive in a set of postulates for logic may be complex as an element in knowledge. The data of epistemology and the primitive ideas of logical systems are not simple in the same sense. For formal logic, the simplest primitive ideas for any system are those which are the smallest in number and which yield by deduction all the propositions belonging to the system; thus a number of different concepts might be chosen as primitive for different sets of postulates which apply, nevertheless, to the same subject-matter. The facts to which these primitive ideas refer might have the appearance of extreme complexity, but this would not make the ideas any the less primitive and simple, so far as the logical part they play in the system is concerned. This is not the sense in which the basic ideas of a theory of knowledge are simple; these ideas are not arbitrarily chosen because of their economy and fruitfullness in deduction. Their simplicity must correspond to a certain simplicity in the subject-matter; what they mean must present the appearance of simplicity, it must appeal to the mind as being capable of no further analysis. It is this sort of simplicity which is lacking both in the concept of negative facts and in that of incompatibility as the basis of negation.

It is possible to dissociate negation from truth and falsity, and thus to gain in completeness of analysis. The incompatibility or logical opposition of a negative and its positive is not the most fundamental idea involved in negation; this notion of incompatibility arises from a combination of the idea of truth with that of negation in a highly general sense. In order to understand a positive proposition or symbolic expression, it is not necessary to consider whether it is true or false. (The only symbols whose

[1] H. M. Sheffer, *A Set of Independent Postulates for Boolean Algebras*, Transc. Amer. Math. Soc., vol. xiv, no. 4, pp. 481–488.

significance involves their truth are the elementary simple symbols from which complex expressions are constructed.) It is sufficient to know the form of the proposition and the meanings of its elements. The understanding of a negative, similarly, ought not to require a knowledge of its truth or falsity, or — what amounts to the same thing — of its incompatibility with another proposition. A negative must be such that it can be entertained as a possibility for thought without any consideration whatsoever of any object that it might mean, or that any other proposition might mean. Some relation which can be directly apprehended in apprehending the significance of propositions must be sought to mark the limits of the possible values of negative expressions. This will yield the more general negation which does not depend on truth and falsity.

III

This relation is that of *distinctness of meaning;* and it is the symbolic counterpart of the *diversity* or *otherness* of objects. A negative proposition means "something distinct from that which is meant by the positive"; and this, coupled with the ambiguity of the reference, is the whole of negation when the question of the truth or falsity of the negative does not enter — that is, when the negative is purely a possibility for thought.

This definition of the possible values of a negative proposition seems to remove all limitations and to lead to startling conclusions. Among the possible values of "the sun is not shining" are thus included, not only propositions that might be the ground of this negation, such as "it is night" and "the sky is overcast," but also propositions that could by no stretch of the imagination be its ground — propositions such as "Chicago is in Illinois," "man is a rational creature, " etc. In short, any proposition which is distinct in meaning from "the sun is shining" is a possible interpretation of "the sun is not shining."

The appearance of paradox is partly removed if it is remembered that the negative proposition does not actually mean one of its possible values. It is a negative only while it is ambiguous; and, though some propositions are relevant to it as grounds from which its truth or falsity might be inferred, it no more signifies a particular one of these than it does a particular one of the countless propositions irrelevant to it as grounds. The statement "the sun is not shining" does not actually mean "Chicago is in Illinois," nor does it mean "the sky is overcast." It means "something other than that the sun is shining." If a pessimist were asked whether life is good, he might reply, "anything but"; and this inelegant, yet emphatic denial correctly indicates the range of possible values of a negative.

It must be observed that the limit of the possible values of a negative is marked, not by the diversity of objects signified, but by diversity of significance, irrespective of objects. On this widest interpretation of the principle of negation, false propositions (which correspond to no objects) may be values of the negatives of false propositions. "New York is not in Brazil" need not signify merely what is meant by some true proposition; it might mean (though being ambiguous it does not actually mean) that "New York is in Turkey." And if I say that "Cinderella did not love her stepmother," which is false because there was no Cinderella, still I might mean that "Cinderella hated her stepmother," which is also false. The question whether or not the truth of a negative precludes the truth of its positive is irrelevant to this purely conceptual negation.

Any use of symbols presupposes distinctness as well as identity of meaning. Thought is not a repeated series of tautologies, and no symbolic system could be constructed from symbols whose meanings were all equivalent. Just as every symbol, if it is to retain its identity (to be the *same* symbol) in its recurrent instances, must be interpreted in one and only one way, so a

symbol, if it is to have anything other than a reference so vague that it could not be called a meaning, must be distinct in significance from some other symbol; and where distinctness of meaning is present, negation is also present. The dictum that "determination is negation" is no less true for thought than for reality. Without negation, that is, without distinctness of meaning, there is no meaning; and without diversity there are no objects. Everything that *is* [1] is diverse from something else, and every significant idea is distinct in meaning from some other idea. Just as the same symbol or concept, when it is applied to the world of objects, will (by the very definition of the *same* symbol) mean one and only one object, so a symbol that is distinct from it will (by the very definition of a *distinct* symbol) mean a different object or no object. And the negative, since it has the same significance as *any* symbol distinct from the positive, will therefore always be distinct in meaning from the positive.

IV

This general condition of the use of symbols — that a negative is always distinct in meaning from its positive — which follows from the purely conceptual definition of the meaning of a negative, is the principle of contradiction in its most abstract form. It is like the principle of identity in that it is a general rule of symbolism, and therefore a general principle of thought.

Now the law of contradiction is often confused with the law of the excluded middle, which states that "either p or *not-p* is true"; but this latter principle has no connection with the principle of contradiction, when truth and falsity are not considered in the definition of negation. As a general rule of symbolism the law of contradiction asserts that to every symbol corresponds another, its negative, whose meaning is distinct from the first.

[1] In the sense of ch. V.

It can be written, "$a \neq$ *not-a*," the sign ($\neq$) being taken to mean "distinct from." Both the laws of identity and contradiction, on their purely symbolic interpretations, state general intentions in the use of symbols. The law of identity, "$a = a$," tells us that any symbol "a" is equivalent, for symbolic purposes, to itself; that it can replace itself in any context without altering the meaning, since every symbol can have one and only one meaning. The law of contradiction, on the other hand, tells us that there is at least one symbol that is distinct, for symbolic purposes, from any given symbol "a," and this is "*not-a*"; if it replaces "a" in any context, the meaning will be completely altered.

As statements of general intentions in the use of symbols, the laws of identity and contradiction are *a priori* in the sense that they are rules laid down by the mind for its own guidance. They are self-affirming; if they are denied, they are assumed, for no proposition or concept can be framed without them. In knowing what it is to think, we know these laws, and we can escape them only by escaping from the use of propositions or concepts — that is, from the use of symbols.

There is another interpretation of these principles, an *existential* interpretation, which construes them as general truths about objects, as well as general principles of symbolism. If "a" stands for an object — for *any* object — "$a = a$" asserts that "any object is identical with itself," or simply that "any object has an identity"; while "$a \neq$ *not-a*" asserts that "any object is distinct from anything other than itself," or simply that "any object stands in the relation of diversity to something." Both of these statements are tautologies, but they could not be true unless there were identity and diversity in the world of fact. They express the minimal conditions of the being of objects — that is, of the objects which are presented or represented in empirical-rational knowledge.

The very assertion of these general truths, however, assumes the laws of identity and contradiction as principles of symbolism. Only if "*a*" is the same symbol as "*a*" does "$a = a$" affirm that "any object is identical with itself"; and unless "*a*" and "*not-a*" are always distinct in meaning, "$a \neq$ *not-a*" might mean "$a \neq a$." The predication of diversity in objects presupposes diversity in the meanings of symbols, apart from objects meant, but the statement that "$a \neq$ *not-a*" in the symbolic sense does not imply that *a* or *not-a* stands for an object. Though it is false to say that "griffins are not not-griffins," if this means that griffins exist as distinct things, this is not false as a statement of how the concepts "griffin" and "not-griffin" are to be used.

If the law of contradiction were not valid both on its symbolic and existential interpretation — if there were no distinct symbols and no diverse objects — thought would find itself in a strange situation. It would be impossible to think of anything without at the same time thinking of everything. In order that a symbol may mean a specific object, not only must the object be self-identical and the symbol be the *same* symbol in its different instances, but there must be something that is not meant by this symbol and that can be signified only through a different symbol — something for the apprehension of which a new and distinct concept is the only appropriate instrument. Without negation thought would be even less definite than the Parmenidean tautology, "Being is"; and without diversity in objects, the whole structure of the world of fact, as we know it, would disappear.

The limiting condition of the reality that gives truth, as we have defined it, to symbolic expressions or propositions is that it must be capable of being consistently presented and represented; and, so, *for this reality*, the principles of identity and contradiction on their existential as well as their symbolic interpretations

must be true *a priori*. Anything that could be consistently presented or represented would necessarily be self-identical and diverse from other things. The principles of identity and contradiction are the "formal" categories of this limited being; the test of consistency, which makes use of the "empirical categories" of science and common sense, can be applied to existent objects only if these more general principles hold of them. These principles are implicit in the condition that marks the limits of this reality.

How much further the validity of the laws of identity and contradiction, as truths about reality, can be extended is a matter for speculation; and here again the theory of knowledge finds its completion in metaphysics. If reality were constituted by thought, these most general conditions of thought would certainly be the most general conditions of being. But for a theory any less extended than this, it is a wide leap from the most general rules of thought to the most general principles of being. It is possible that reality cannot be trimmed to the neat dimensions of thought; there may be nooks and corners of the real hidden from this knowledge which demands the identity and diversity of its objects. It is even possible that these very requirements of thought form a barrier between the mind and reality, confining knowledge to a realm of Kantian phenomena. But neither this metaphysical alternative, nor the other — that they are true *a priori* of reality in the most inclusive sense of the term — can be established on the premises so far laid down.

V

What of the third "law of thought," the principle of the excluded middle?

To construe the meaning of a negative proposition as "anything other than the meaning of the positive" appears to be a direct violation of this law, for this permits a false proposition to

be a value of the negative of a false proposition and therefore allows the possibility that when p is false *not-p* might also be false. "Paris is not the principal city of England" might mean "Paris is the capital of the United States," and this is clearly not allowable if the principle of the excluded middle is true. Moreover, "Paris is not the principal city of England" could mean (though on account of its ambiguity it does not actually mean) that "birds have wings" or that "to-day is Friday," and the truth or falsity of these propositions seems to be totally unconnected with the truth or falsity of "Paris is the principal city of England."

The law of the excluded middle, that "either p or *not-p* (but not both) is true," connects negation with truth and falsity by adding a further condition than diversity of meaning to determine the possible values of a negative. We thus return to the more complex and special type of negation, which Mr. Demos defined in terms of incompatibility. This law is not a necessity of thought in the same sense as are the principles of identity and contradiction in their most general forms. The latter are needed if concepts or symbolic expressions are employed in any way whatsoever; the former enters only when truth and falsity (as well as the identity and distinctness of concepts) are considered. It is necessary for inference — for the derivation of the truth or falsity of propositions from the truth or falsity of others; but it is not necessary for the very existence of meaning, as are the first two laws of thought.

In order that the principle of contradiction may yield the law of the excluded middle, the negative must be interpreted in a special way, which takes account not only of the distinctness of concepts, but of their truth.

Now true propositions are distinct through the objects they mean, as well as distinct in significance apart from these objects; but false propositions are not distinguished from one another

through objects meant, since they mean no objects. Moreover, any true proposition is distinct from any false proposition through an object meant, for the one means an object, while the other does not. This notion of "being distinct through an object meant" supplies the necessary principle for determining the possible values of a negative, when the law of the excluded middle is added to the laws of identity and contradiction. If *not-p* means (ambiguously) the same as any proposition that is distinct from *p* through the distinctness of an object meant, it will follow that when *p* is true *not-p* is false, and *vice versa*, that when *p* is false *not-p* is true.

To include the idea of *an object meant*, that is, of truth, in the definition of negation is to remove negation from the realm of the purely conceptual. It is no longer sufficient merely to know the meaning of a proposition — merely to consider it as a possibility for thought — in order to know the meaning of its negative. One must also know whether it is true or false. But if this kind of negation is less simple than the other, it is more effective; negatives now become instruments of inference. All false propositions fall together as indistinguishable through an object meant, though they still remain distinct as concepts; and hence the negative of a false proposition can mean only true propositions. It will be true for any value that can be chosen for it, so that the truth of *not-p* can always be inferred from the falsity of *p*. "New York is not in Brazil" will not include among its possible interpretations, "New York is in Turkey" or "New York is in Australia," since these propositions are not distinct through an object meant from "New York is in Brazil." Any possible value of "New York is not in Brazil" will be a true proposition. Among these possible values there will be one that is the most probable ground of this negation, *viz.*, "New York is in the United States"; but it is not necessary to suppose that this value is the sole possible meaning of the negation in order to

establish its truth. Any true proposition will be a possible value of this negative.

On the other hand, if *p* is true — and if there is a single false proposition among all the propositions which can be formulated — then *not-p* will be false. For among the possible values of *not-p* will be included, not only all the true propositions that are distinct from *p* through an object meant, but also some false proposition; and since the negative is ambiguous, it might mean this false proposition. It cannot be true if there is a single possibility of falsity.[1] Thus, since it is true that "carbolic acid is poisonous," and since some false proposition can be stated — let us say the proposition that "carbolic acid is good to drink" — the negative, "carbolic acid is not poisonous," might mean that "carbolic acid is good to drink." This latter proposition, being false, is a possible value of the negative of the original true proposition, for any true proposition is distinguished from any false proposition by an object meant. And though the negative, "carbolic acid is not poisonous," might also mean countless other propositions, both true and false, it cannot be true so long as there is any possible interpretation for which it is false. Thus where the possible values of a negative are the meanings of those propositions which are distinct from the positive through an object meant, the falsity of a negative can always be inferred from the truth of its positive.

It is clear that this more restricted type of negation — this *inferential* negation — eliminates possibilities which the wider and simpler form of negation permits. When the negative is construed in this narrower way, "fairies do not live on bread and meat" could not possibly mean "fairies live on dew and honey." Its positive being false (since there are no fairies), this propo-

[1] The assumption that some false proposition can be formulated amounts to the assumption that there is falsity; and certainly a negative could not be false unless there were falsity. There must be some false proposition that it could mean.

sition must mean the same as some true proposition; it must be true. By the very definition of the meaning of the negative, we are no longer permitted to consider it as a mere possibility that "fairies do not live on bread and meat"; we are forced to infer its truth from the falsity of the proposition of which it is the negative. The principle of the excluded middle thus drags us down to earth from the world of pure speculation.

VI

The conditions of the truth and falsity of negatives have already been mentioned in passing. A negative is false if there is a single false proposition among its possible values, and it is true only if all of its possible values are true propositions. This follows from the ambiguity of the negative. Let us examine this idea more closely.

Now the meaning of a proposition determines whether it is true or false, and if this meaning is not fully specified — that is, if the proposition might mean a number of different things, but actually does mean none of these — the truth of the proposition will depend on what it *might* mean, not on what it actually does mean; for it actually does mean no specified object. Thus an ambiguous proposition (or one which contains ambiguous constituents) is not true in the same sense as an unambiguous proposition. Its truth, like its meaning, is fixed by possible references to objects, rather than by an actual reference. It is "ambiguously true." To be true in this sense is to be susceptible of many different interpretations, but *not to be susceptible of a false interpretation*. Just as ambiguous meaning is an unspecified possibility of meaning, so ambiguous truth is an unspecified possibility of truth, rather than truth for a definite object or class of objects. And if among the possible determinations of such an ambiguous expression there is *one* for which it would be false — if there is a *single* possibility of falsity — the proposition fails of truth as

an ambiguous proposition, even if it might be true for some of its possible values. Though an ambiguous proposition, if it is asserted, need not be asserted for *all* of the things it might mean, its truth requires that it be capable of being true for *all* of these.

Consider the statement, "I saw a friend on the street yesterday." This is a particular proposition but, since it contains an ambiguous constituent, it does not assert that I saw a particular friend — Brown or Jones; nor does it assert that I saw *all* my friends. I might have seen any one of them, and if among them I count, as did the poet Blake, a certain group of friendly spirits, or if some whom I call friends are not friends, the proposition might be false; and the mere possibility of falsity invalidates it as an ambiguous assertion. If the assertion is true — which means that all possibility of interpreting it as false is eliminated — any person that could be meant by the variable "a friend" must be a real person, and must be really a friend.

A negative is "ambiguously true." If it is asserted, it is not asserted for one of its possible values or for all of them; it is ambiguously asserted. And if it is true, it must be true for more than one or several possibilities; there must be no possibility for which it could be false. A true negative embraces the entire universe of true propositions, but not one false proposition; while a false negative embraces a universe of true and false propositions —one true proposition, only, being excepted: the positive of this negative. If these conditions of the truth and falsity of negatives are born in mind, the remaining implications of the principle of the excluded middle are easily seen to be true; *viz.*, that when *not-p* is false, *p* is true; and when *not-p* is true, *p* is false.[1]

Suppose that *not-p* is false. Then some proposition that could be a value of it, some proposition other than *p*, must be false.

[1] This principle, "either *p* or *not-p* is true (but not both)," implies that (1) if *p* is false, *not-p* is true, (2) if *p* is true, *not-p* is false — both of which have been demonstrated in the previous section — (3) if *not-p* is false, *p* is true, and (4) if *not-p* is true, *p* is false.

But if *p* is also false, *p* would not be distinguished "through an object meant" from this value of *not-p;* that is, *not-p* would not be the negative of *p*. And therefore *p* must be true. Thus, since it is false that "men do not desire pleasure," it cannot also be false that "men do desire pleasure." For, if this were the case, one of the possible values of the negative, "men do not desire pleasure" — let us say the false proposition "men desire pain" — would fail to be distinct from the positive, "men desire pleasure," through an object meant; so that "men do not desire pleasure" would not be the negative of "men desire pleasure." The very definition of the possible values of this negative demands that its positive be true when it is false.

On the other hand, when *not-p* is true, *p* must be false. For, *not-p* being true can mean only true propositions; and, since it is always possible to state some false proposition (since there is always falsity), unless *p* were false when *not-p* is true, *not-p* would include this false proposition among its possible values, for this false proposition would be distinct from *p* through an object meant, if *p* were true. Therefore *p* must be false. It is true, for example, that "fortune is no respecter of persons"; but some false proposition, *q*, can be stated; and if it were also true that "fortune is a respecter of persons," this latter proposition would be distinct from the false proposition, *q*, through an object meant, and *q* would be a possible value of its negative. The negative might then be false. Thus it must be false that "fortune is a respecter of persons" if it is true that "fortune is no respecter of persons."

Where the negative means ambiguously the same as any proposition distinct from the positive through an object meant, the law of the excluded middle is therefore valid in all of its implications. From the truth (or falsity) of *p*, the falsity (or truth) of *not-p* can always be inferred; and from the truth (or falsity) of *not-p*, the falsity (or truth) of *p* can always be inferred.

VII

The examination of judgment led us to the conclusion that to disbelieve or deny a proposition is to believe or assert its negative — belief, like understanding, being an attitude of preparation for or anticipation of something meant. But if the meaning of a true negative might be that of any one of an entire universe of true propositions, what possible attitude of preparation or anticipation could be appropriate to it? To prepare for so many truths would be to prepare for — and to believe — no specific truth; so that the analysis of negation seems to have given negatives a basis in fact at the cost of rendering their significance so vague that they cannot be believed.

And yet there is a kind of belief which is appropriate to negative propositions in their total ambiguity — a belief that is directed toward no specific object or class of objects. Such a belief is an anticipation of anything at all other than what is denied; it is a general readiness to accept something or other — anything excepting the proposition disbelieved — as true. This is the frame of mind in which the atheist denies God; he is prepared to believe anything but that there is a God. And it is the attitude one takes toward propositions which are plainly self-contradictory or false. If you tell me that "triangles have four sides" or that "oxen fly in the air," I shall probably reply that I can accept anything but this. This kind of disbelief, however, though it has a positive content of a formal and indefinite sort, amounts to little more than turning the mind away from the proposition disbelieved. It is a *bare* (and often groundless) disbelief which hardly escapes scepticism and which, like the sceptical attitude, can never take the place of a positive judgment in knowledge.

Negatives are rarely thus asserted or believed in the full extent of their possible meanings. Ordinarily, values which are relevant as *grounds* to the falsity of the proposition disbelieved

are selected from among the possible values, and on these belief centers.

Now the ground of a negative — and of the assertion of the falsity of a proposition — is some proposition which is incompatible with the proposition negated or with the proposition that is believed to be false. It is a proposition which is never true when the other is true. And this incompatibility is determined by the "empirical categories" of science and common sense. Scientific theories and laws, and prescientific generalizations, are the tests not only of the existence of objects and the truth of propositions, but also of the "non-existence of objects" and the falsity of propositions. A proposition is false — it stands for no object and is conveniently, if inaccurately, said to mean a "non-existent object" — when it is inconsistent with the whole of knowledge. The mere failure to discover something in existence that the proposition might mean, though it may be ground for denial, is no proof of the falsity of the proposition, and can lead only to the bare disbelief which is directed toward no specific truth.

The grounds of negative judgments thus fall into two classes: the failure of verification(which is really privation of ground), and the discovery of true propositions from which the falsity of the proposition denied can be inferred by the aid of the generalizations of science and common sense. In the first case, when I believe a proposition to be false, I can merely deny it by affirming its negative in its full ambiguity. In the second case, my belief will be capable of some limitation; I can select as its special objects the truths from which the falsity of the proposition in question follows. I am not therefore always confined to the formal knowledge that "anything other than what is denied is true," in affirming the falsity of propositions, though this is all that the definition of the meaning of the negative can tell me. I can infer, through the laws which I have found to hold in experi-

ence, that certain propositions are true when the other is false, and I can direct my belief toward these propositions. These are the "most probable" values of the negative. If I assert that "no man defies the laws of society with impunity," I most probably believe that "crime is always punished" or that "criminals are detected" or that "the social outcast is miserable," any one of which could be the ground of this negative judgment. But since there are, in any case, many possible grounds of falsity,— many propositions which are incompatible with the one negated,— to believe a negative, even when a whole universe of truths is not considered, is to prepare for more than a single contingency. And yet it is to prepare for a contingency, to be ready to act as if some definite propositions were true; and thus disbelief becomes as fully positive as it is capable of becoming.

The fact that this more fully positive type of disbelief is directed toward specific ones among the possible values of the negative does not restrict the *meaning* of the negative to these values, that is, to its possible grounds. The negative does not cease to be an "infinite negative" (as the older logicians called it), which might mean the same as any one of an indefinite number of propositions; nor are the formal requisites of its truth — that it cannot be false for any possible interpretation — altered.

Thus negative judgments are never wholly equivalent in effect to positive ones, for there is always a margin of possibility which is not covered by belief, even when the "most probable" values of the negative are selected as the objects of belief. The "infinity" of the negative allows for the totally unforeseen and unforeseeable — for contingencies which none of the principles of science and common sense embrace. When one believes a negative, therefore, he can find in his belief a residuum of that general readiness for anything other than what is denied — a margin of "preparation for the wholly unexpected" — which arises from

the ambiguity of the negative. Negative judgments, no matter what their grounds, remain evasive to the end; they commit one to much less than positive judgments. They do not completely attain to the solidity and certainty of positive judgments.

VIII

It must not be forgotten that ideas can be entertained apart from belief and disbelief, as possibilities for thought; that negation can be dissociated from truth and falsity; and that there is a realm of the purely conceptual, where identities and distinctions of meaning alone are relevant.

The fact that the laws of identity and contradiction (when truth is not a part of the definition of the negative) are general rules of symbolism and therefore of thought, irrespective of objects which are or might be meant, makes it possible to distinguish *conceptual validity* from *truth*, and formal consistency from existential consistency. A system is conceptually valid, whether or not it is true, if it does not violate these two principles of the use of concepts. This means that symbols which are originally taken as distinct — to be values of the negatives of one another — must continue to be treated as distinct, and that symbols originally taken as identical must continue to be treated as identical. These conditions are sufficient to give the system formal consistency. Whether it is existentially consistent and true — that is, whether it contains no incompatible propositions — is another matter, but this will in no sense affect its formal consistency. The consistency of the system is a consistency of definition only.

This sort of formal consistency and conceptual validity belongs to uninterpreted deductive systems and to the whole world of romance and fiction. The canons of the constructive imagination are the laws of identity and contradiction, where these laws are rules of symbolism and nothing more.

IX

The present analysis of negation, beginning with Mr. Demos's notion that a negative proposition means ambiguously the same as some positive proposition which is incompatible with the proposition negated, has separated the idea of negation from that of truth, thus distinguishing purely conceptual from *inferential* negation. The law of the excluded middle requires that the negative be construed in the latter sense, and the law of contradiction in its most general form, that it be construed in the former.

The assumption of "negative facts" as the referents of true negative propositions does not analyze negation; nor does the theory that the negative means ambiguously some proposition incompatible with the positive carry the analysis to its completion, for the notion of incompatibility is complex; and though *incompatibility* might be taken as simple for purposes of deductive exposition, it is not simple as an element in knowledge. Incompatibility is a function of the truth and falsity of propositions. Negatives are unquestionably ambiguous, but if the possible values of a negative are restricted to its possible grounds, that is, to those propositions whose truth excludes the truth of the positive, the meaning of a negative cannot be dissociated from its truth or falsity.

A negative proposition, where the negative is interpreted in the simplest way, means (ambiguously) the same as any proposition distinct in meaning from the positive, and this distinctness of meaning need not be connected with diversity in objects meant, that is, with truth. Distinctness of meaning, apart from objects referred to, is a necessary presupposition of the use of symbols; and the principle of contradiction, "$a \neq$ *not-a*," as a general rule of symbolism and hence as an *a priori* condition of thought, asserts that a negative is always distinct in significance

from its positive. This principle is coördinate with the law of identity, "$a = a$," which asserts that any symbol is equivalent (or identical) in meaning with itself. Both of these principles can be interpreted in another way, that is, as general conditions of the *being* of objects, and these existential interpretations assume the principles as rules of symbolism. If a means an object — *any* object — the law of identity states that "any object is identical with itself," and the law of contradiction that "any object is distinct from anything other than itself." These statements are true only if existing objects have the formal characters of identity and diversity. For the limited reality which can be consistently presented and represented — for the *existing objects* of empirical-rational knowledge — these laws must hold; but whether they hold for all reality can be determined by nothing less than a complete metaphysical theory.

The third law of thought, the principle of the excluded middle, joins the idea of truth to that of negation. This principle is not necessary to the very existence of meaning, as are the others, but it is necessary to inference. If the negative means the same as any proposition distinct from the positive *through an object meant*, it will always be true that "either p or *not-p* is true (but not both)."

To believe a negative in its full ambiguity is to be ready to accept "anything other than the positive," which is disbelieved. This form of bare denial does not limit itself to the possible grounds of the falsity of the proposition disbelieved. More commonly, denial and disbelief are directed toward values specially selected from among the possible values of the negative; that is, toward the values which might be grounds of the falsity of the proposition denied. These grounds are determined by the generalizations of science and common sense, which are the tests of the "non-existence of objects" and the falsity of propositions, as well as of the existence of objects and the truth of proposi-

tions. But no negative judgment is wholly equivalent in effect to a positive one, since its ambiguity is never eliminated, and it might mean something other than it is believed to mean.

The material consistency of the knowledge whose truth and falsity is tested by its compatibility or incompatibility with other portions of knowledge (accepted as true) can be contrasted with the formal consistency of conceptual systems which conform to the principles of identity and contradiction, as general rules of symbolism, but are not affirmed to be either true or false. Conceptual validity and truth are distinct. It is possible to construct valid deductive systems which are completely uninterpreted, in which the identity and distinctness of concepts (or symbols) and of logical forms alone are considered. Logical form can be isolated and studied in itself. This is the work of the purely formal deduction which, in Mr. Russell's phrase, "does not know what it is talking about, or whether what it says is true."

CHAPTER VII

FORMAL DEDUCTION

I

FORMAL logic might be defined as "the science of pure form," for it is interested in forms, and their connections, apart from any "matter" in which they might be exemplified. Since symbols in themselves embody logical forms, formal deductive systems can be presented in uninterpreted symbols, they can be stated as if they referred to no objects; and logical form, thus dissociated from a "matter," becomes a subject of study in itself.

There are numerous sets of postulates for geometry, — both Euclidean and non-Euclidean, — for arithmetic and algebra, for serial order, and for logic itself, which attain — or nearly attain — this ideal of abstract statement, and it is these systems which best illustrate pure deduction. An examination of them makes it plain that formal deduction is concerned with nothing but forms and their relations. In order that the deductive connections in these systems may stand forth clearly, they are separated from the subject-matter and stated in terms of postulates, axioms, and theorems which might equally well apply to other subject-matters. Though these postulates, axioms, and theorems can be interpreted in many different ways, their form remains the same whatever objects they refer to — even if they refer to no objects. Their form is independent of all interpretations.

Such systems are hypothetical in the sense that they are purely conceptual and not asserted. They conform to the principles of the construction of concepts, that is, of the use of symbols, and are therefore possibilities for thought; but the question of their truth or falsity does not enter. If complete interpretations

for them can be found, they will be "materially" consistent and true for these interpretations; the hypothetical connections between the postulates, axioms, and theorems will become inferences; the theorems can be truly asserted as consequences of the postulates and axioms. But if they have no interpretations, or only incomplete interpretations, these systems will still be possibilities for thought — conceptually "valid" and "formally" consistent; and, from the point of view of formal logic, this is all that they need be.

It is not possible to empty a set of postulates of all meaning, for it would then cease to be a set of postulates. An uninterpreted deductive system is not a collection of meaningless marks. It has its plan of syntax, its groups, its terms and symbols of unity; its meaning is syntactical; within a general framework of significance, the uninterpreted symbols become variables, which, though they do not actually refer to objects, *could* refer only to objects whose formal characteristics are those given in the symbolic groups of the system. The general scheme of symbolic structure determines the possible values of these variables.

Some symbolic context is always necessary to a variable. Outside a context, a variable is not a symbol (and not a variable); and though the context of the variable may include symbols of fixed significance, this context need be only a formal plan of grouping. If every element, excepting the form, of a symbolic expression is a variable, the expression will not lose its meaning. If only one factor in the meaning is determinate, *viz.*, the form, this will be sufficient to give the expression significance as a whole — a significance that has reached the maximum of indeterminateness. It is then the use of variables in a general context of logical structure which makes the study of form in itself possible.

The meaning of a formal deductive system is syntactical, and so the postulates of the system are a plan of syntax, which de-

termines what groups have significance in the system and how these groups can be derived from one another. The deductive connections appear as rules of substitution, by which one symbol (or group) can replace other symbols (or groups) and yield expressions which still have meaning. To state this plan of syntax, it is necessary to assume nothing but the general principles of symbolism and certain possibilities of substitution; these assumptions are sufficient to make the system a connected whole which embodies a number of different logical forms related (hypothetically) to one another as premises to conclusions. The structure alone is constant; all other elements in the meaning are variable. When the system is asserted for a definite (and complete) interpretation, these hypothetical connections become the premises and conclusions of inferences, and the propositions that were assumed in their maximum generality, without interpretation, prove to be a deductive exposition of truths which hold for certain sets of objects. But when there is no interpretation, the system retains its syntactical meaning; through the conditions of significance imposed on it, it continues to be a presentation of logical forms connected in certain ways; and this is all it originally purports to be.

It is not strictly true that, in putting forward a formal deductive system, "we do not know what we are talking about," though there is no doubt, since all the elements of the system with the exception of the form are variable, that we need not know or care "whether what we say is true." We are talking about logical forms, which are given in the very symbols through which we talk about them. We are mapping *possible* systems of concepts, as Columbus mapped the continent of which he discovered only the shore; but in this case it is not even necessary to know that there is a continent to be mapped; we need know only the principles of map-making to explore the world of possibilities.

II

The general principles of symbolism, that is, the most general conditions of the construction of concepts, have already been stated. They are (1) the principle of group significance, that the meaning of any group is a function of the meanings of its elements and their grouping; (2) the general rule of "symbolic grammar," that every significant group must contain both a symbol of unity and terms (or a term), and this rule demands that the symbols of unity be distinguished from the terms; (3) the principle of identity, that every symbol has one and only one meaning; and (4) the principle of contradiction, that thereis at least one symbol which is always distinct in meaning from any given symbol, that is, its negative, which means (ambiguously) the same as any symbol other than the positive; and this principle demands that there be distinctness as well as identity of meaning.

Every symbolic system must be such that it *could* be interpreted, even though it refers specifically to no objects; otherwise, it would not be a possibility for thought. The general principles of symbolism themselves are the conditions under which systems could be interpreted. From these principles, in the above order, it follows that any purely conceptual system must, first, be composed of groups which are taken to be significant as wholes. If the system contained no such groups, its symbolic elements would not be variables, but mere meaningless marks.[1] Thus there must be certain *signs of syntax* to indicate the structure of the groups — their divisions into major and minor wholes, and the group relations between these wholes.

Secondly, the symbols of unity must be distinguished from the terms; and when the system is interpreted, the former will stand

[1] Or else symbols with a determinate reference to objects, in which case the system would not be uninterpreted.

for relations, operations, or qualities in their rôle as elements of unity in facts, while the latter will stand for individuals or for universals in their substantive rôle. There are no objects which are not terms related or qualified or operated on in some way; hence, without this distinction between symbols of unity and terms, the system could not be interpreted and would not be a possibility for thought.

Thirdly, every symbol must be identical in meaning with itself, that is, the equation, $a = a$, must hold throughout the system; and this principle (the principle of identity) gives a rule of substitution which is valid in all systems, *viz.*, the rule that any symbol can be substituted for itself without altering the meaning of the group in which it is substituted. The concept of identity makes it possible, moreover, to define symbols which are distinct in character as identical in meaning. Indeed, many of the symbolic elements of a conceptual system may enter the system only through being identical in meaning with the original groups of other elements, that is, many of the variables may gain their variable significance through *definition* alone. Such equivalent symbols can replace one another in any of the groups; this also is a rule of substitution which is valid in all systems.

Fourthly, symbols originally taken as distinct in meaning must be construed as distinct throughout. None of the transformations of the system can result in the use of distinct symbols as the same or equivalent. For this would violate the principle of contradiction, which asserts that any possible value of a negative, that is, any symbol distinct from a given symbol, is *always* distinct from this symbol. And yet this principle *permits* the substitution of distinct, as well as identical, symbols for one another, when the special rules of the system so provide. Though such substitutions of distinct symbols for one another will always result in alterations of meaning, this fact does not render such substitutions impossible, for distinct symbols do not become

identical in meaning by being substituted for one another. Identity of meaning is more than a possibility of mutual substitution.[1] In fact, the more important transformations in any system are the ones which come about when the symbols in groups are replaced by other (distinct) symbols. Special rules of substitution, which hold only for the special systems in question, provide for transformations of this type.

A formal set of postulates will begin by assuming that certain symbolic groups, whose terms and symbols of unity have been distinguished, are significant, that is, that these groups have syntactical meaning apart from any objects they might represent. A set of postulates will also assume some special rules of substitution, in addition to the general rule that the same symbol or symbols of identical meaning can replace one another. It may further assume certain special identities of meaning, that is, special equivalences or definitions, which (when the system is interpreted) will represent identities in objects. With these assumptions, the set of postulates is a complete plan of syntax for a symbolic system. Any group or equation which can be derived from the original groups or equations by the rules of substitution will be significant in the system, and combinations of symbols which cannot thus be derived will be nonsensical; they will have no meaning in this context. The plan marks the boundaries of an area of concepts; it separates what is conceivable or significant in terms of its own symbols from what is inconceivable or nonsensical in these terms; but it does not separate what is true from what is false. All of its propositions are "thinkable"; whether they are true or false is another and an irrelevant question. Deduction in a system of this type is the manipulation of the symbols according to the rules.

[1] Symbols which are identical in meaning must, if they stand for objects, stand for the same object; while symbols which can be substituted for one another may stand for diverse objects, and must stand for diverse objects if they are distinct in meaning.

The process of formal deduction, divorced from a subject-matter and occupied only with logical forms and their connections, is the derivation *by substitution* of significant symbolic groups of different elements and forms from other symbolic groups; and a formal deductive system is a set of uninterpreted symbolic complexes of many distinct forms and elements built up, by this process of substitution, from a few simple symbols and symbolic complexes. Where possibilities alone are considered, deduction is an engrossing game played with symbols. The rules of the game are the general conditions of the use of symbols, together with the possibilities of manipulation opened up by these rules and by any other special rules which may be devised for special systems.

III

The study of a set of postulates — for geometry, algebra, logic, or any other field of knowledge — reveals the importance of the part which substitution plays in formal deduction. Logicians speak of substitution as an essential *modus operandi* or *real* operation in uninterpreted systems.[1] It is in fact the sole means of connecting the theorems with the postulates and axioms when these postulates and axioms are purely hypothetical, that is, when their truth or falsity as premises and conclusions is not even considered.

Sets of postulates rarely state their rules of substitution explicitly, though they always make use of such rules. One of the simplest methods of providing for the manipulation through substitution of the symbols of a deductive system is that employed by Mr. E. V. Huntington.[2] As the "base" of a set of postulates, he assumes a class, K, of elements, a, b, c, etc., and

[1] See C. I. Lewis, *A Survey of Symbolic Logic* (1918), pp. 353 ff.; also A. N. Whitehead, *A Treatise on Universal Algebra* (1898), Bk. I, ch. 1, "On the Nature of a Calculus."

[2] See E. V. Huntington, *Sets of Independent Postulates for the Algebra of Logic*, Transc. Amer. Math. Soc. (1904), v, 288–309.

relations or operations, R, S, etc. Since the postulates hold for *any* element of K, the terms, a, b, c, etc., can be mutually substituted for one another without rendering the postulates invalid. Thus, if (aRb) is postulated in the system, (aRc) and (bRc) will also be in the system. Further, if the group (aRb), is itself an element of the class, K, it can be substituted for a, b, or c (since it might be *any* element of K), giving the complex $((aRb)Rb)$, etc. And if the substitutions were carried forward — (aRb) replacing a or b in this latter group, and so on — an infinity of complexes of different forms would be provided for by this single possibility of substitution. The result of assuming the class, K, as a "base" is then the same as if it were explicitly stated, in the beginning, that any of the simple symbols of the system can be mutually substituted for one another, and any of the original groups can be substituted for any of the simple symbols.

In the *Principia Mathematica*, Messrs. Whitehead and Russell make use of substitutions in most of their proofs, especially in those of the earlier propositions from which the later ones follow by implication; and they speak of this process as "noticing that (these propositions) are instances of general rules," [1] that is, of the primitive propositions given in the first section of their work. These substitutions are possible, in the symbolic system of the *Principia Mathematica*, because these primitive propositions are principles of logic asserted for *any* proposition, rather than purely uninterpreted groups of symbols. In Mr. Huntington's language, the class, K, of the *Principia Mathematica* is the class of *all propositions;* any proposition can be substituted for any other in the original complexes of the system. For example, $p \vee p \cdot \supset \cdot p$, states that "any proposition *or* the same proposition implies itself." And from this postulate, together with the definition of implication, $p \supset q \cdot = \cdot \sim p \vee q$, which asserts that "an implication between two propositions

[1] Whitehead and Russell, *Principia Mathematica*, i, 102.

means that either the first is false or the second true," the principle of *reductio ad absurdum*, $p \supset \sim p \cdot \supset \cdot \sim p$ ("any proposition that implies its negative implies its own falsity"), can be derived by substitution as follows: The proposition, $\sim p$, (*not-p*), is substituted for p in the primitive proposition, $p \vee p \cdot \supset \cdot p$, giving $\sim p \vee \sim p \cdot \supset \cdot \sim p$. Now, if $\sim p$ is substituted for q in the definition of implication, the result is $p \supset \sim p \cdot = \cdot \sim p \vee \sim p$; and if $p \supset \sim p$ (being equivalent to $\sim p \vee \sim p$) is substituted for the latter in $\sim p \vee \sim p \cdot \supset \cdot \sim p$, we have $p \supset \sim p \cdot \supset . \sim p$, the principle of *reductio ad absurdum*, which was to be proved. This proof illustrates how the *Principia Mathematica* derives the "immediate consequences of the primitive propositions." "The recognition that a certain proposition is an instance of some general proposition previously proved or assumed," say Messrs. Whitehead and Russell, "is essential to the process of deduction from general rules";[1] and this is, in effect, the recognition that certain substitutions are permitted in the system.

It must be observed that these deductive substitutions are always carried through completely. If one symbol replaces another in a group, it replaces the latter wherever it occurs. Thus, when $\sim p$ is substituted for p in $p \vee p \cdot \supset \cdot p$, every instance of the symbol p becomes an instance of $\sim p$, giving $\sim p \vee \sim p \cdot \supset \cdot \sim p$. That the substitutions must be complete is a general principle of deductive substitutions, the reason for which is not immediately evident.

It will be remembered that the distribution of tautologous (or recurrent) elements is a formal feature of symbolic groups. Now, if the rule of *completeness of substitution* were not followed, groups that include tautologous members could be transformed into groups in which no tautologous members were present. A reflexive group, such as (*Raa*), could yield a non-reflexive group, such as (*Rab*). There is no objection to this if the system in ques-

[1] *Loc. cit.*

tion contains a non-reflexive group (which is otherwise the same in form) for every reflexive group.[1] But if the system contains no non-reflexive groups (or only non-reflexive groups which do not correspond in other formal features with its reflexive groups), the rule of completeness of substitution will be needed; otherwise, complexes which have no significance in the system will be introduced. In the first type of system (one which contains a non-reflexive group corresponding to every reflexive group), it is always possible on the other hand to derive the reflexive from the non-reflexive groups by substituting instances of the same symbol for distinct symbols: (*Rabc*), for example, gives (*Raaa*) if *a* is substituted for both *b* and *c*, so that the rule of completeness of substitution can be imposed on these systems without decreasing their deductive possibilities. The rule is therefore assumed as a general principle, applicable alike to those systems for which it is necessary and to those for which it is redundant. The reason is, in short, that non-reflexive complexes are more general in form than reflexive ones. The rule of completeness of substitution prevents us from deriving the more general forms (the non-reflexive) from the less general (the reflexive), but permits us to derive the less general from the more general by substituting instances of the same symbol for distinct symbols.

Though this latter type of substitution — of instances of the same symbol for distinct symbols — is *permitted* by the rule of completeness of substitution, it is not *required* by this rule, and it cannot be assumed that in all systems instances of the same symbol can replace distinct symbols. If this were erected into a general principle, there would be no systems which did not include reflexive groups, and there are many such — notably, systems of serial order. Indeed, for some systems, it can be laid down as a special rule that distinct symbols must always be re-

[1] The term "reflexive group" is here taken in a general sense to mean any group containing tautologous members, however distributed.

placed by distinct symbols. In these systems it would be impossible to derive a complex such as (*Raaa*) from one such as (*Rabc*). This special rule prevents the introduction of tautologous elements where tautologous elements were not originally present; it is applicable to systems whose complexes are irreflexive throughout. But if this narrower condition of substitution is not introduced, it is taken for granted that instances of the same symbol (that is, tautologous symbols) can replace distinct symbols; that reflexive groups can be derived from all the non-reflexive groups in the system.

In most deductive systems, then, the rules of manipulation permit any of the original terms mutually to replace one another, and any of the basic or derived groups to replace any of these terms — the rule of completeness of substitution being always observed, and the rule that distinct symbols must be substituted for distinct symbols being sometimes added.[1]

Still other possibilities of manipulation than these highly general ones are usually provided for in a set of postulates. It may be specified that some of the groups are identical in meaning with others or with some of the original terms, and hence that these symbols of identical meaning can be mutually substituted for one another. In Mr. Huntington's first set of postulates for the algebra of logic, among other equations, the following appears: $a \oplus (b \otimes c) = (a \oplus b) \otimes (a \oplus c)$. When the system is interpreted, this equation means "a or b and c is identical with a or b and a or c." [2] It represents different structures in an identical object, that is, on the class interpretation of the algebra, the expressions on each side of the sign of equality stand for the same class. But when the system is uninterpreted, the equation merely states an intention to use these groups as identical in

[1] There are some systems, however, in which the basic or derived groups cannot be substituted for the original terms, and an example of such a system is given below. See below, sec. vii, note.

[2] See E. V. Huntington, *op. cit.*

meaning, whether or not they refer to objects. Since such assumed identities of meaning equate different logical forms, they are "equations of structure."

In lieu of or in addition to these equations of structure, a system may include connections of meaning which appear as implications. It may be specifically provided that some of the groups *imply* others. For instance, in Mr. Huntington's postulates for serial order, it is stated that (aRb) (bRc) "implies" (aRc), and this asserts (where R stands for the relation "precedes") that "if a precedes b and b precedes c, a precedes c." [1] Now, in an uninterpreted system, these cannot be implications of the usual sort, for ordinarily implications are functions of the truth values of the propositions they unite; p implies q means that "either p is false or q is true," that is, that when p is true, q is also true. Truth values are not considered in an uninterpreted system, and so these implications cannot be truth-implications. They become truth-implications when the system is interpreted. Otherwise, they are connections of meaning apart from truth and falsity, and as such are nothing but *possibilities of substitution*. If p *implies* q in an uninterpreted system, this implication means simply that q can replace p *as a conclusion* in any deduction. Any series of transformations that leads to p leads also to q.

These "implicational substitutions" belong to a special class, and must not be confused with the others. If q is an "implicational substitute" for p, this provision does not allow q to take the place of p in any complex in which p appears; but it does allow q to be substituted for p when p appears at the end of a chain of deductions. And this is readily verified by any concrete example of implication: "x is human" implies "x is mortal," and if "x is human" is deduced from the fact that "x walks erect and speaks," "x is mortal" can also be deduced from this fact. But "x is mortal" cannot replace "x is human" wherever

[1] E. V. Huntington, *The Continuum* (1917), p. 10.

the latter occurs; for instance, "x is human" implies that "x thinks"; and yet "x is mortal" does not imply that "x thinks." An implicational substitute for any expression cannot replace this expression as a premise, or as a constituent in any expression whatsoever, but it can replace it as a conclusion.

There is one other important type of manipulation which must be noticed. The process of substitution is transitive, that is, if a can be substituted for b and b can be substituted for c, a can be substituted for c. If each of a series of propositions can replace one another in order, all of the intermediate steps can be dropped; the first of the series can be replaced by the last. This transitivity of substitutions enables us to bring together the premise and conclusion of a long line of deductions, the intervening links being omitted. And this principle applies where the substitutions proceed *via* identities of meaning, or where a conclusion is replaced by something that it implies. Such *condensations* of chains of deduction are employed in all systems; and the general rule that permits them will hereafter be used without specific statement, and will be referred to in proofs as "condensation."

The single *modus operandi* of pure deduction is the "real operation" of substitution. By this operation alone the theorems of a deductive system are developed from the basic groups, equations, and implications; and by this operation, also, the premises and conclusions of the system are brought together or "condensed." The postulates and theorems affirm no truths and no relations between truths. All that is considered is what groups have meaning in the system, and how these groups can replace one another.

The manner of formulating a plan of syntax, that is, a set of postulates, for a deductive system, so that the basic groups can be transformed by substitutions into theorems, will be better understood through a more complete illustration, in which there

are many different possibilities of symbolic manipulation. The substitutions will always follow (1) the rule of completeness and (2) the rule that the same symbol, or symbols of identical meaning, can replace one another; but the special condition, that distinct symbols must be substituted for distinct symbols, will not be imposed. When this system is before us, we can ask what connections among facts the deductive manipulations of the symbols might represent.

IV

The system chosen as an illustration is a Boolean Algebra, an algebra of logic; and it will be interpreted as referring to classes and class relationships, though it can be interpreted in several other ways, *e.g.*, in terms of regions in space together with certain relations between these regions.[1] But no interpretation is necessary to it. Through symbolic complexes of variable elements, which have syntactical meaning apart from objects they might mean, it presents an area of possible logical forms and shows how, from a few of these forms, all the others can be derived. Logical forms and their connections are its sole concern. It is complete in itself as a collection of symbolic groups, equations, and rules of manipulation; and if it had no interpretation it would still be an experiment in possibilities for thought. That it happens to be a class algebra is an interesting and useful accident, but not essential to it.

(1) The basic groups of this system are (*Rab*), (*Sab*), and (*Na*). *R*, *S*, and *N* are undefined *symbols of unity*, and *a* and *b* are undefined *terms*. The parentheses are *signs of syntax* or grouping, to be construed in the way in which such signs are always construed. (Since these groups have a logical structure, they are significant, and the symbols *R*, *S*, *N*, *a*, and *b* are vari-

[1] See E. V. Huntington, *op. cit.*

ables.) A third undefined term, c, will also be assumed. R, S, N, a, b, and c are symbolically distinct.[1]

(2) The substitutions permitted are: (a) any of the undefined terms can be *mutually* substituted for one another, and (*b*) any of the basic symbolic groups, or any groups derived from them, can be substituted for any of the undefined terms. This latter rule permits substitutions in one direction only, that is, the undefined terms cannot be substituted for any of the groups, unless they are equivalent to them. (The rule of completeness of substitution, and the principle that the same or equivalent symbols can replace one another, are taken for granted.)

(3) The following identities of meaning are assumed:

A. $Ra(Sb(Nb)) = a$.
B. $Sa(Rb(Nb)) = a$.
C. $Ra(Sbc) = S(Rab)(Rac)$.
D. $Sa(Rbc) = R(Sab)(Sac)$.

Since the original undefined terms of the system appear on both sides of these identities, these identities are not definitions. When the system is interpreted, these equivalences will represent differences of structure in an identical object.[2] They are "equations of structure."

(4) Two symbols which are not among the undefined terms are introduced by definitions:

E. $z = Sa(Na)$.
F. $u = Ra(Na)$.

These are definitions because z and u are not included in the original terms but enter the system only through being identical in meaning with certain ones of its significant groups. Such definitions are not necessary to the exposition; they are merely symbolic conveniences which condense complex expressions. But it would be uneconomical to take as undefined, and hence to

[1] The spatial order of the symbols in the groups is irrelevant; it plays no part in their logical form, which is determined by grouping alone; and therefore it is not necessary to mention the fact that (Rab), (aRb), and (abR) are the same symbol.

[2] See above, ch. II, sec. vi.

place among the original (primitive) symbols of the system, a symbol whose meaning could be defined through some of the groups. "Occam's razor" must always be applied; entities must not be assumed unless they are necessary. (It is to be noted that the rules of substitution given in (2) hold for the original terms, and not for z and u. There are special rules for these terms, which can be deduced from the other rules.)

The general and special conditions of substitution permit three classes of transformations in this system: (1) The elements of a group can be altered without altering its form. If, for example, a replaces b, and c replaces a, in $Ra(Sb(Nb)) = a$, this substitution will give the equation, $Rc(Sa(Na)) = c$, which is of the same form as the first, but of different elements. Such transformations are unimportant. (2) Tautologies can be introduced where tautologies did not originally appear, for there is no proviso that distinct symbols must be replaced by distinct symbols. Thus from $Ra(Sb(Nb)) = a$, we can derive $Ra(Sa(Na)) = a$; from (Rab) or (Sab) we can derive (Raa) or (Saa), etc. The system therefore contains reflexive forms corresponding to all its non-reflexive forms. (3) Complexes of a higher type can be derived from complexes of a lower type, and this process can be carried, theoretically, to infinity. (Rab), for example, yields $(Ra(Rab))$, and this complex yields $(Ra(Ra(Rab)))$, etc., when (Rab) is substituted for b; so that the system contains an infinity of possible forms.

It is apparent that all of the complexes which appear in the equations, A, B, C, and D, and in the definitions, E and F, are significant in the system; they can all be derived from the basic groups by these rules of substitution.

The "equations of structure" and the definitions, moreover, make possible a fourth class of transformations, those obtained by substituting equivalent symbols or groups *in* groups. Without the "equations of structure," each of the groups in the sys-

tem might, when the whole is interpreted, stand for a distinct object. The equations impose limitations on these possible distinctions of meaning, as do also all the other equations which can be deduced from them by proper substitutions; and at the same time these equations increase the number of manipulations which can be performed in the system.

(5) Some of the most important theorems, and their derivations, are the following:

1. $Raz = a.$ 2. $Sau = a.$

The proof of these two theorems requires the definitions, E and F. These definitions are, like the other equivalences in the system, subject to all the permitted transformations by substitution. Therefore, $z = Sa(Na)$ yields $z = Sb(Nb)$ by the substitution of b for a; and $u = Ra(Na)$ yields $u = Rb(Nb)$, by a similar substitution. And, since equivalent symbols can be substituted for one another, we can derive $Raz = a$ from the equation $Ra(Sb(Nb)) = a$ by replacing $Sb(Nb)$ by z. Similarly, we can derive $Sau = a$ by replacing $Rb(Nb)$ by u in the equation $Sa(Rb(Nb)) = a$. This is the deduction of theorems 1 and 2.

The rules for the substitution of these two special symbols, z and u, follow from their definitions. Since they are equivalent, respectively, to $Sa(Na)$ and $Ra(Na)$, they can be substituted for any symbol for which these groups can be substituted. Therefore, they can replace any of the terms, a, b, c, etc., of the system; but these terms *cannot* replace them, for these terms cannot replace the groups through which z and u are defined. In other words, so far as concerns the manipulation of the symbols in this system, z and u are values of any of the variable terms, but they are not themselves variables; they are constants.[1]

[1] This introduces a new kind of variability, "functional variability," which is something other than indeterminateness of significance; z and u are functional constants, rather than functional variables; but their meaning is still undetermined. This is explained below, sec. vi.

Other theorems are the following:

3. $Raa = a$

Proof:

$a = Sau$	(by theorem 2).
$Raa = Su(Raa)$	(Raa replaces a in 2.)
$Su(Raa) = S(Ra(Na))(Raa)$	(By def. F, $Ra(Na)$ replaces u).
$S(Ra(Na))(Raa) = Ra(Sa(Na))$	(where a replaces b, and Na replaces c, in C).
$Ra(Sa(Na)) = Raz$	(By def. E, z replaces $Sa(Na)$).
$Raz = a$	(by theorem 1).
Therefore: $a = Raa$	("condensation").

4. $Saa = a$.

This can be deduced in exactly the same manner as 3, by using theorem 1 for 2 and u for z. Theorems 3 and 4 are the analogues of one another for the symbols of unity R and S respectively.

5. $a = N(Na)$

Proof:

1'. $z = S(Na)(N(Na))$	(by def. E, where Na replaces a).
2'. $u = R(Na)(N(Na))$	(by def. F, where Na replaces a).
$a = Raz$	(by theorem 1).
$Raz = Ra(S(Na)(N(Na)))$	(By 1', $S(Na)(N(Na))$, replaces z).
$Ra(S(Na)(N(Na))) = S(Ra(Na))(Ra(N(Na)))$	(by C, where Na replaces b and $N(Na)$ replaces c).
$S(Ra(Na))(Ra(N(Na) = Su(Ra(N(Na)))$.	(By def. F, u replaces $Ra(Na)$).
$Su(Ra(N(Na))) = Ra(N(Na))$	(by theorem 2, where $Ra(N(Na))$ replaces a).
Therefore: 3'. $Ra(N(Na)) = a$	("condensation").
$N(Na) = R(N(Na))z$	(by theorem 1, where $N(Na)$ replaces a).
$R(N(Na))z = R(N(Na))(Sa(Na))$.	(By def. E, $Sa(Na)$ replaces z).
$R(N(Na))(Sa(Na)) = S(R(N(Na))a)(R(N(Na))(Na))$	(by C, where $N(Na)$ replaces a, a replaces b, and Na replaces c).
$S(R(N(Na))a)(R(N(Na))(Na)) = S(R(N(Na))a)u$	(By 2', u replaces the group $R(N(Na))(Na)$).
$S(R(N(Na))a)u = R(N(Na))a$	(by theorem 2, where $R(N(Na))a$ replaces a).
Therefore: 4'. $N(Na) = R(N(Na))a$	("condensation").
Therefore: $N(Na) = a$	(by "condensation" of 3' and 4').

When the system is interpreted as a Boolean Algebra, this theorem is the principle of double negation. (It is to be noted in the proof that the group $Ra(N(Na))$ in 3′ is the same as the group $R(N(Na))a$ in 4′, since the spatial order of the symbols is irrelevant.)

6. $z = Nu$

Proof:

$z = Sa(Na)$	(by def. E).
$z = Su(Nu)$	(u replaces a in def. E).
$Su(Nu) = Nu$	(by theorem 2, where Nu replaces a).
Therefore: $z = Nu$	("condensation").

7. $u = Nz$

Proof:

$u = Ra(Na)$	(by def. F).
$u = Rz(Nz)$	(z replaces a in def. F).
$Rz(Nz) = Nz$	(by theorem 1, where Nz replaces a).
Therefore: $u = Nz$	("condensation").

Some of the other theorems which can be derived in the system, but of which the proofs are not given, are the following:

8. $Rau = u$	12. $Rab = N(S(Na)(Nb))$
9. $Saz = z$	13. $Sab = N(R(Na)(Nb))$
10. $Ra(Sab) = a$	14. $R(Rab)b = Ra(Rab)$
11. $Sa(Rab) = a$	15. $S(Sab)b = Sa(Sab)$

The introduction by definition of the group $(b(Ia))$ simplifies the system; on the class interpretation of the algebra this group means that the class a is included in, or "subsumed under," the class b.[1] This new complex can be defined in several different ways:

$(b(Ia)) = (Rab = b)$	(G)
$(b(Ia)) = (Sab = a)$	(H)
$(b(Ia)) = (Rb(Na) = u)$	(I)
$(b(Ia)) = (Sa(Nb) = z)$	(J)

[1] The grouping of $(b(Ia))$ indicates that it has a "sense" or order; the alternative order is $(a(Ib))$. No specific symbol of unity is inserted to join the sub-group (Ia) or (Ib), as the case may be, to the term a or b; the outer parentheses show that they form a group, and this is all that is necessary; their group unity is of the most general sort — simply "togetherness" or "unity."

Some of the postulates and theorems stated above can be translated into the terms of this new group, and this translation will give, among others, the following theorems:

16. $(a(Iz))$

Proof:

$(Rbz = b)$ is in the system	(b replaces a in theorem 1).
$(Rab = b) = (b(Ia))$	(by def. G).
$(Rzb = b) = (b(Iz))$	(z replaces a in def. G).

Therefore: $(b(Iz))$ is in the system, since it is equivalent by definition to a group in the system.

17. $(u(Ia))$

Proof:

$(Rau = u)$ is in the system	(by theorem 8).
$(Rab = b) = (b(Ia))$	(by def. G).
$(Rau = u) = (u(Ia))$	(u replaces b in def. G).

Therefore: $(u(Ia))$ is in the system.

18. $(u(Iz))$

Proof:

$(Ruz = u)$ is in the system.	(u replaces a in theorem 1).
$(Ruz = u) = (u(Iz))$	(u replaces b, and z replaces a, in def. G).

Therefore: $(u(Iz))$ is in the system.

19. $(a(I(Sab)))$

20. $((Rab)(Ia))$

The proof of theorems 19 and 20 is similar to that of 16, 17, and 18, where theorems 10 and 11 and definitions G and H are employed.

21. $(a(Ia))$

Proof:

$(Raa = a)$ is in the system	(by theorem 3).
$(Raa = a) = (a(Ia))$	(a replaces b in def. G).

Therefore: $(a(Ia))$ is in the system.

There are many more theorems in terms of the group $(b(Ia))$ which are not stated.

This group is unlike the groups in terms of R, S, and N in that there are no rules which permit the substitution of $(b(Ia))$, or

its derivatives, for the original terms, a, b, and c, of the system. The group is defined as identical in meaning with the equivalence ($Rab = b$), or with the other equivalences given as H, I, and J. It can, therefore, be substituted for these equivalences; and when the latter, or their derivatives, are members of the system, some I-groups will also be, by definition, members of the system. But ($Rab = b$), or the other equivalences, H, I, and J, cannot be substituted for a, b, and c; and hence the I-groups cannot be substituted for these terms.[1]

V

In practice, all deductive systems are devised with one eye on the facts, that is, on an interpretation. They are stated *as if* they referred to no objects, but they prove in the end to be connected expositions of truths which hold in some realm of experience, that is, to be systems of geometry, algebra, logic, etc. And if the plan of a system were constructed without this *arrière pensée*, it is not likely that an interpretation — or anything more than a trivial one — could be found for it. To explore all possibilities for thought, cut off from moorings in the world of the actual, would be an interesting but an endless experiment, for the number of possible conceptual systems is infinite; and so those systems which embody the structure of actual sets of objects are selected. Pure speculation then gives way to assertion, what seem to be arbitrary manipulations of symbols become inferences, and the significance of the whole is bound down to the real. A conceptual system which corresponded to no subject-matter would be an exercise in logical construction, and nothing more. Yet such exercises show that from the point of view of

[1] The original rule of manipulation, that any of the basic groups or their derivatives can replace any of the basic terms, does not provide that the "equations of structure" or the definitions, taken as wholes, can replace these terms; for these identities of meaning are not derived from the basic groups (though their constituents are), but are postulated on their own account.

formal logic all that need be considered, even in an interpreted system, are logical forms and their connections.

The system which has been developed here is adapted to several different subject-matters. The symbols a, b, and c can stand for propositions, and the groups (Rab), (Sab), and (Na) for certain "truth-functions" of propositions; *viz.*, (Rab) will mean the proposition, "either a or b (not excluding both) is true"; (Sab) will mean the proposition, "a and b are both true"; and (Na) will mean the negative of a. Another interpretation can be given in terms of regions of a plane surface, together with certain relations between these regions; *viz.*, (Rab) represents the region that includes both regions a and b; (Sab) represents the region where a and b intersect; and (Na) represents all of the plane that lies outside the region a.[1] We shall interpret the system as a logic of classes.

A class has previously been described as a totality of distinct objects which have some predicate in common. Every predicate determines such a totality, *e.g.*, the predicate "man" determines the class "men." Classes may be related to one another in many ways, and these class relationships can be employed in reasoning. Classes may overlap, they may include or exclude one another, their members may be added together to form a new class. There is also a null class, one that has no members, the class of "nothing"; and this class is determined by any predicates of which there are no instances, *e.g.*, "the present King of France" determines the null class. Further, there is a universal class of which everything, excepting this class itself, is a member.[2] Every postulate and theorem of the system can be inter-

[1] See E. V. Huntington, *Sets of Independent Postulates for the Algebra of Logic*, cited above.

[2] This exception is necessary to avoid the difficulties involved in the notion of "the class of all classes." See Whitehead and Russell, *Principia Mathematica*, ch. 2. Though the universal class is not a *member* of itself, still it is coextensive with itself; *i.e.*, the relation of class inclusion or subsumption, represented by the I-groups of the system, holds for it; so that $(u(Iu))$ is true.

preted as a true statement about classes, and every manipulation as a true inference, leading from premises to conclusions that concern classes.

The original terms of the system, *a*, *b*, and *c* stand for any classes; but since these terms are symbolically distinct, they can not, in any expression, be interpreted as signifying the same class. (*Rab*) means the class that includes all the members of *a* and *b*, and only these members; this is called the "logical sum" of *a* and *b*. (*Sab*) means the class whose members belong both to *a* and *b*; it is the "logical product" of *a* and *b*. (*Na*) means the class of all objects excluded by the class *a*. The symbol *z* stands for the null class, and *u* for the universal class, for "nothing" and "everything," respectively. The expression (*b*(*Ia*)) means that the class *a* is included in the class *b*.

The rules of substitution tell us:

(1) That any of the original terms can be mutually substituted for one another, and that any of the basic groups, or groups derived from them, can be substituted for any of the original terms. Now if, on the class interpretation, (*Rab*), (*Sab*), (*Na*) and their derivatives represent anything, they will stand for classes. The logical sums, products, and negatives of classes, together with their derivatives, are themselves classes. Hence they can replace the variables *a*, *b*, and *c* in any expression and the expression will still be true, for *a*, *b*, and *c* are *any* classes. Moreover, every sum and product *might* be the sum or product of a class and itself; so that every expression containing distinct symbols can be altered, by substitutions, into one containing tautologous symbols without rendering the expression untrue. (For the reason that groups formed by *R*, *S*, and *N* stand for the same general sort of entities, that is, classes, as those meant by their terms, these symbols of unity represent *operations*, rather than relations.) [1]

[1] See below, sec. vii.

(2) The group $(b(Ia))$, which means that the class a is included in the class b, cannot, on the other hand, be substituted for a, b, and c, since it does not stand for a class. If a is included in b, the classes a and b are *related*, but this relation constitutes a new sort of fact, which is not itself a class.

(3) The classes z and u can replace any of the original terms, a, b, and c, but these terms cannot replace z and u; for the null and universal classes are special elements, they are unique. What is true of *any* class is true of them, since they are classes, but the converse does not hold.

(4) Equivalent groups or symbols can replace one another, for they represent the same class. Thus the null class is defined by and can be substituted for $Sa(Na)$; it is the logical product of any class and its negative, that is, it is the class that includes all the members both belonging to and excluded by any class a — and these are no members.

Obviously, these rules of manipulation make it possible, through the substitution of interpreted symbols or groups in other interpreted groups, to derive true propositions about classes from other true propositions about classes; they yield true inferences.

The interpretations of the postulates are not so simple as those of some of the theorems. Theorem 1, $Raz = a$, is the familiar proposition that "the logical sum of any class and the null class is identical with the first class," that is, "the class including all members of a and of the null class is the same as a." Theorem 2, $Sau = a$, states that "the class of members common to a and the class of *everything* is the same as a." Definition E, $z = Sa(Na)$, is the principle of contradiction for classes, and asserts that "a class and its negative have nothing in common." Definition F, $u = Ra(Na)$, is, on the other hand, the law of the excluded middle for classes; it affirms that "the class including all members of a and all members excluded by a is the class of

everything," or simply "that a class and its negative exhaust the universe." Theorems 3 and 4, $Raa = a$ and $Saa = a$, are two forms of the principle of identity for classes. The first states that "the class including all the members of a and of a is the class a"; and the second, that "the class of members common to a and a is the class a." Theorem 5, $a = N(Na)$, that is, "the class that excludes all that is excluded by any class a is the class a," is the principle of double negation; "*not-not-a* is identical with a." Theorems 6 and 7, $z = Nu$ and $u = Nz$, show that the universal and null classes exclude, or are the negatives of, one another, that is, "nothing" excludes "everything" and "everything" excludes "nothing."[1] Theorem 8, $Rau = u$, states that "the universal class added to any class a gives the universal class"; and this theorem differentiates the u (and the "addition") of the class algebra from the number 1 (and the "addition") of ordinary algebra; 1 added to any number does not give 1. Theorem 9, $Saz = z$, however, shows an analogy between the null class and the zero of numerical algebra: "the logical product of any class a and the null class is the null class." Theorems 10 and 11 are two forms of the principle of "absorption": $Ra(Sab) = a$ asserts that "any class a absorbs by addition a logical product of itself"; and $Sa(Rab) = a$, that "any class a absorbs the sum of itself and another class in a logical product of itself and this sum." Theorems 12 and 13 are forms of the principle of "transposition": $Rab = N(S(Na)(Nb))$ allows a sum of classes to be transposed into the negative of the product of the negatives of these classes; while $Sab = N(R(Na)(Nb))$ allows a logical product of classes to be transposed into the negative of the sum of the negatives of these classes. Theorems 14 and 15 are the "associative" law for class sums and products, respectively. Postulates C and D are the "distributive" law; $Ra(Sbc) = S(Rab)$

[1] By theorem 18, "nothing" is also subsumed under or included by "everything." The null class has the peculiar property of implying its own negative.

(*Rac*) states that "the sum of a class and a product of classes is the same as a product of two sums" (loosely put); while *Sa*(*Rbc*) = *R*(*Sab*) (*Sac*) states that "the product of a class and a sum of classes is the same as a sum of products" (loosely put). The symbols express this proposition more clearly than a brief form of words can, as is also the case with postulates A and B: *Ra*(*Sb*(*Nb*)) = *a* asserts that "if the logical product of a class and its negative is added to another class, the whole will be identical with this latter class"; and *Sa*(*Rb*(*Nb*)) = *a* asserts that "if the logical sum of any class and its negative forms a product with another class, the whole will be identical with this latter class."

The "commutative" law, *Rab* = *Rba* and *Sab* = *Sba*, does not appear in this system. It is unnecessary to state that "the logical sum or product of *a* and *b* is the same as the logical sum or product of *b* and *a*" for the reason that the spatial order of the symbols in the groups is not relevant to their significance. *Rab* is the same symbol as *Rba*, and *Sab* is the same symbol as *Sba;* the operations of class addition and class multiplication are symmetrical, and these symbolic groups — which are without order — represent them as such.

The group (*b*(*Ia*)), however, has an order; "the class *a* is subsumed under the class *b*" means something different from "the class *b* is subsumed under the class *a*." The order (*b*(*Ia*)) stands for the first of these facts, and the order (*a*(*Ib*)), for the second; and these differences of order are represented by the distribution of the terms *a* and *b* in the two constituent groups of the expression (*a*(*Ib*)), not by the spatial arrangement of the symbols.

The theorems in terms of *I*-groups, interpreted as class inclusions or subsumptions, state the following propositions: Theorem 16, (*a*(*Iz*)), affirms that "the null class is included in any class," that is, any class covers as great a logical area as a class of no members, and, if it is not itself a class of no members, covers

a wider area. Theorem 17, $(u(Ia))$, expresses the fact that "any class is included in the class of *everything*," which is plainly true. Theorem 18, $(u(Iz))$, is, like theorem 16, paradoxical; it asserts that "the null class is included in the class of *everything*," that is, that the universal class covers everything which is covered by the null class—and much more. Theorem 19, $(a(I(Sab)))$, states that "any class includes the members common to itself and another class"; and theorem 20, $((Rab)\ (Ia))$, that "any class is included in the class formed by itself and another added together." Theorem 21, $(a(Ia))$, tells us that "any class includes and is included by itself." The definitions of class inclusion, G and H, are clearly true: $(b(Ia)) = (Rab = b)$ says that "if a class b is identical with the class composed of all the members of a and b, this is equivalent to the statement that b includes a"; and definition H, $(b(Ia)) = (Sab = a)$, says that "if a class a is identical with the class whose members are common to a and b, this is equivalent to the statement that a is included in b."

All of the original postulates, definitions, rules of substitution, and many of the theorems of this deductive system are thus seen by inspection to be true for classes. But in order to verify the system as a whole for this interpretation, it is necessary to verify only the postulates, definitions, and rules of substitution, for if these original propositions hold for classes, all the propositions derived from them will hold for classes. The reason for this will be apparent from a further consideration of the meaning of rules of substitution in general, and of the inferences to which they lead when they are interpreted.

VI

Symbolic groups built up from other symbolic groups or from simple terms are *functions* of these groups or terms. (We have observed before that the idea of a group is the same as that of a function.) Every symbol in an uninterpreted deductive system

has a *functional range*, which is simply all the groups into which it can enter as a constituent. A rule of substitution lays down conditions determining the functional ranges of symbols; it tells us either that the functional ranges of two symbols coincide or that the one is wider than the other, that is to say, it prescribes what their functional ranges shall be. If it were possible to spread out on a single page all the groups of a symbolic system, there would be no need for rules of substitution; we could see at a glance what was the functional range of every symbol. But in all except trivial systems the number of possible groups is very great, usually infinite, and so rules of substitution are necessary.

If a symbol x can be substituted for a symbol y in an uninterpreted system, the functional range of x is wider than that of y, that is, for every group in the system containing y as a member there is a group (similar in every other respect, both in its constituents and in its form) containing x as a member, but not the reverse. And if x and y can be mutually substituted for one another, their functional ranges coincide, that is, there are no groups in the system containing x which are not paralleled by groups (similar in every respect except for the presence of x) containing y, and vice versa.

This conception of the functional range of a symbol introduces a new kind of variability, *functional variability*, which is wholly different from the interpretational variability — indeterminateness of reference — hitherto considered. A functional variable is a variable with respect to the groups in which it can play a part; and objects, no less than symbols, exhibit this type of variability. Just as a symbol composed of other symbols is a symbolic function, so an object composed of other objects, that is, a factual complex, is an objective function. We describe objects through their predicates and relations; any object may have many different predicates and relations, it may enter into

many different complexes; and its functional range is all the complexes in which it can be a member. With respect to these groups the object is a variable; it may appear now in one group and now in another, and other objects may take its place in groups. Thus a deductive system, when it is interpreted, will describe a certain set of objects through certain selected functions of these objects.

Strangely enough, the variables of wider functional range are the *values* of the variables of narrower range; the former are the more specific in meaning, while the ones of narrower range are the less specific. Thus, in the system now under consideration, the terms z and u enter in functions in which the terms a, b, and c cannot play a similar part; z and u are functional variables of wider range than a, b, and c, and hence are special values of the latter. (Theorem 6, $z = (Nu)$, for example, cannot be translated into $z = (Na)$ or $a = (Nu)$.) But the reason for this is evident. The more we know *about* an object, the more determinate it becomes for thought, and the more functions in which a symbol enters, the more completely is its rôle in the system fixed.

A rule of substitution, then, lays down the conditions of functional variability for the interpretational variables of a symbolic system. In doing so it prescribes the form of the system and at the same time, through prescribing the form, imposes limitations on the interpretation. When the symbols are given (interpretational) values, these values must be such that the facts into which they enter are of the same structure as the symbolic groups into which the uninterpreted symbols enter; and even so, a wide range of possible (interpretational) values is permitted to any of the symbols of the system. But the interpretation must always fulfill the conditions imposed by the rules of substitution, and this requires that the objects meant by any symbol or group must have a functional range in the world of fact which corresponds to the functional range of this symbol or group in

the uninterpreted system. This being the case, the manipulations of the symbols will always yield true propositions.

Thus, if the terms a, b, and c, in the Boolean Algebra stand for classes and the symbols of unity R, S, and N signify the operations of class addition, multiplication, and negation, it will be possible to substitute a, b, and c for one another for the reason that any class forms a sum or product with any other class, and also has a negative. The groups (Rab), (Sab), (Na), and their derivatives can be substituted for these terms, for these groups also signify classes (complex classes) and so can have sums, products, and negatives of their own. Every complex class enters into the same sort of class operations as every simple class. But the simple terms a, b, and c cannot replace the symbols for these complex classes for the reason that there are certain functions of complex classes which are not functions of simple classes.

In the interpreted system, the rules of substitution cover a multitude of *existential* or "material" *implications* which hold for the objects meant. The rule which permits the substitution of (Rab), (Sab), (Na), for the terms a, b, and c informs us that "if there are classes, there are sums, products, and negatives of these classes"; the provision that a, b, and c can be mutually substituted for one another tells us that "if there are functions of one class, there are similar functions of other classes." But these existential implications must not be confused with specially postulated, and uninterpreted, implications of the sort mentioned before, which permit the substitution of certain symbols for others as *conclusions* in chains of deduction. When the system is interpreted, these specially postulated connections of meaning will be "formal" rather than existential implications. Thus, though (Na) yields by substitution (and hence existentially implies) the complex $(N(Na))$, this could not mean that (Na) "formally" implies $(N(Na))$, for this would assert that

"any negative implies its own negative." The substitution really means that "the negative of a negative is an instance of a negative," or "if there is a negative of a class, there is also a negative of this negative." [1]

A symbol whose range of functional variability includes but does not coincide with that of another is a constant with respect to this variable of narrower functional range. The symbols z and u in the Boolean Algebra are constants with respect to the terms a, b, and c, since z and u can be substituted for these terms, but not the reverse. Yet z and u are not completely determined in significance by this fact; there are still as many possible interpretations for them as there are for the system. A symbol whose meaning is variable in interpretation may function as a constant, when its part in an uninterpreted system alone is considered. Such a symbol is a "functional constant," and is known in mathematics as a parameter. Moreover, all the groups of our system are, with respect to the terms a, b, and c, functional constants; any one of these groups stands for some particular sort of combination of classes. Indeed, if the system is taken merely as an uninterpreted set of symbols, the only elements in it that function as variables are the terms a, b, and c. Every other element or expression is a value of these variables. Therefore, a, b, and c, which have the narrowest range of functionality in the system, are the "functional variables"; and each of the substitutions of a group or other element (unless it be of the elements

[1] The notion that a rule of substitution is to be interpreted as stating a relation between the functional ranges of two variables makes it clear why symbols of indistinct meaning, *i.e.*, the symbols on either side of the sign of identity, can be mutually substituted for one another. When the system is interpreted, these symbols must always be given an identical value, they must stand for the *same* object; and obviously every object has the same functional range as itself. Further, it is clear that a coincidence of functional ranges, and hence a possibility of mutual substitution, is not the same as identity of meaning.

a, b, or c) for a, b, and c, puts a functional constant in place of one of these functional variables.[1]

VII

The symbols of unity R, S, and N stand for *operations*. That they must stand for operations, and not relations, is determined by the deductive use made of the groups in which they appear. The distinguishing feature of an operational group is that it can be substituted *in itself* for one of its terms; and in the Boolean Algebra this is provided for by the rule that (Rab), (Sab), and (Na) can take the place of a in any group, and hence in these very groups. In other words, an operational group is a value of the functional variables that enter into it.

The "plus," "minus," and "times" of numerical algebra, for instance, are operations because the complexes they form are numbers. If a and b represent numbers, $(a - b)$ will be a number; it may be the number a, where b is *zero*, or the number $(-b)$, where a is *zero*. But it will always be some number, and thus it can take the place of a or b in any algebraic expression and still give a number; it can be substituted in the very combination $(a - b)$, giving $((a - b) - b)$, which is a number.

A relational group, on the other hand, is not a value of the variable terms that appear in it. If a and b are points, and (Rab) means "a follows b," this fact is not also a point, and it cannot take the place of a or b in any complex of points. For this reason, the I of the Boolean Algebra is a relation rather than an operation. The I-groups cannot be substituted for the terms that enter

[1] It is possible to consider these "constants" as functional variables of different orders. What is a functional constant with respect to one symbol might be a functional variable with respect to another. The *order* of variability of the symbols is shown by the rules of substitution. Thus, in the Boolean Algebra (Rab) is a functional constant with respect to a and a functional variable with respect to $(R(Rab)b)$, which is a special case of it. But for purposes of simplification, all the groups can be lumped together as constants with respect to the terms a, b, and c.

into them, for if a class a is included in a class b, this does not form a class, as does the logical sum or product or negative of a and b.

Operations give rise to systems of an infinite number of possible forms; an operational group can replace again and again a term which appears in this group itself, yielding each time a group of higher type. For every complex that contains a, in the Boolean Algebra, there is one of higher type that contains (Rab), (Sab), or (Na); so that a "proper part" of the collection of groups in which a is a member can be put into one-to-one correspondence with the whole of this collection.[1] And this proves that these groups are infinite in number.[2]

[1] For a definition of an infinite collection, see E. V. Huntington, *The Continuum*, pp. 7 ff.

[2] A set of postulates for serial order of the simplest sort illustrates the difference between operations and relations, and also shows how a system wholly different from the Boolean Algebra is constructed:

(1) Assume that a, b, and c are distinct symbols for terms; and that the system is built on the basic group $(a(Ob))$ v $(b(Oa))$. (The symbols (v) and (.), which stand respectively for the logical ideas "or" and "and," are used in the statement of the plan of this system; and if the system is to be *completely* uninterpreted, these symbols might be replaced by symbols that do not suggest these logical ideas, *e.g.*, by L and N.) It will be observed that the grouping of the complex $(a(Ob))$ shows that it has an order, irrespective of the spatial arrangement of the symbols; and that $(b(Oa))$ is the alternative order of this asymmetrical form.

(2) The symbols a, b, and c can be mutually substituted for one another; but *distinct symbols must always be replaced by distinct symbols.*

(3) The special "formal" implication, $(a(Ob)).\ (b(Oc))$ "implies" $(a(Oc))$, holds in the system.

It is apparent that $(a(Oa))$ or any combination in which this group might appear is not in the system; for the provision that distinct symbols must always be substituted for distinct symbols will not permit us to derive $(a(Oa))$ from $(a(Ob))$ or from any of the other O-groups. These O-groups are therefore *irreflexive.* Further, the group $(a(Ob))$ is, by its grouping, distinct from $(b(Oa))$; and this is a distinction of sense, *i.e.*, of the distribution of the terms in the constituent groups. The O-groups are therefore *asymmetrical* for distinct terms. (Obviously, if there were an O-group of indistinct terms, *e.g.*, $(a(Oa))$, this would be symmetrical; but there is no such group.) The implication $(a(Ob)).\ (b(Oc))$ "implies" $(a(Oc))$, shows that the O-groups are *transitive;* but transitive only when a, b, and c are distinct, for $(a(Oa)).\ (a(Oa))$ "implies" $(a(Oa))$, and $(a(Ob)).\ (b(Oa))$ "implies" $(a(Oa))$ cannot be derived from this implication without violating the rules of substitution. The fact that the O-groups cannot be substituted for the terms a, b, and c shows that they are

VIII

The apparently arbitrary manipulations of the symbols of an uninterpreted system are "deductions" for this reason: that, when the system is interpreted, that is, when the postulates are *asserted* for a set of objects, these manipulations become *inferences*. The theorems are then asserted on the strength of their connection with the postulates; the truth of the postulates strictly proves the truth of the theorems.

Inference is allied to assertion and belief. There is no such thing as a hypothetical inference, just as there is no such thing as a hypothetical belief. If a proposition is believed, it is removed from the realm of the hypothetical; it is no longer an idea merely entertained, but is accepted as true or, it may be, "ac-

relational, rather than operational, groups; and since $(a(Ob))$ v $(b(Oa))$ is significant in the system, the terms a and b (or a and c, and b and c, if these are properly substituted in this expression) are *connected* in one *sense* or the other by the relation O. This is the postulate of "connexity."

Now, a relation which is connected, irreflexive, transitive for distinct terms, and asymmetrical for distinct terms, is a "serial relation." (See E. V. Huntington, *op. cit.*, pp. 11 ff.) If a, b, and c stand for points on a line, and $(a(Ob))$ means that "the point a precedes the point b, in the order left to right," the postulates can be interpreted as follows:

(1) Asserts that "any point precedes any other point, or this latter precedes it." The condition that the points must be distinct is added by the second half of the rule of substitution under (2).

(2) The first half of this rule of substitution asserts that "if there is a point, there are also two other points"; and the second half asserts that "if any point precedes a point, these two are distinct," since only symbols that refer to distinct points can appear in the complex "a precedes b."

(3) Asserts that "if any point precedes a second, and this second precedes a third, the first point precedes the third."

That the proposition $(a(Ob)) \,.\, (b(Oa))$, *i.e.*, "a point a precedes point b *and* this latter point also precedes a," does not hold in the system is proved by the fact that in order to derive this group from the similar group in the implication, $(a(Ob)) \,.\, (b(Oc))$ "implies" $(a(Oc))$, we must assume a contradiction; we must suppose that a is a distinct symbol from a. With this supposition, a can be substituted, according to the latter half of rule (2), for c in this implication, giving $(a(Ob)) \,.\, (b(Oa))$ "implies" $(a(Oa))$. But if $a \neq a$, this is a violation of the principle of contradiction, which is always assumed as a general rule of symbolism.

cepted as false," which means that its negative is accepted as true. And similarly, a proposition that is inferred from another is accepted as true (or false), not because it is verified in itself, but because it is believed to be connected in a certain way with a proposition previously accepted as true (or false). Inference is the passage from a belief, by way of a belief, to a belief, and is expressed by the *assertion* of a premise and of a connection between this premise and a conclusion, followed by the *assertion* of the conclusion. The connections on which inferences rest are implications. (There are, however, certain types of inference — probable or inductive inferences — which do not rest on the strict implications considered here.)

It is characteristic of true propositions that they can stand alone as objects of belief. Though most propositions have their "reasons," that is, their premises, by which they are implied, any truth can be severed from its premises and believed on its own account. Its truth does not consist merely in the "reasons" that can be given for it. If it is true, it is equally true apart from the propositions which imply it, equally true with or without "reasons." If it were necessary to remember all the logical connections of a proposition believed, belief and assertion would be difficult undertakings; but fortunately the premises of a belief can be forgotten without destroying the truth of the proposition believed (if it is true). Therefore inference, since it leads from one proposition believed to be true to another believed to be true, is well described as "the dropping of a true premise . . . the dissolution of an implication." [1]

But a premise which is not believed cannot be dropped, and for this reason the assertion of a chain of implications is not an inference. "p implies q" means that *whenever* p is true, q is true; but it does not assert the truth or falsity of p or of q; it merely excludes the possibility that "p is true and q false." So far as its

[1] Whitehead and Russell, *Principia Mathematica*, p. 9.

constituent propositions are concerned, a true implication is still hypothetical; though a connection between p and q is asserted, p and q themselves are still entertained and not believed. And if the *supposition* that the premise of this implication is true is added, this supposition will not result in an inference to the truth of q; it will yield nothing but the *supposition* that q is true "if p is true and p implies q." One might go on forever asserting implications and supposing that their premises are true without performing an inference. The chain of implications is broken only when the premises, as well as the implications themselves, are asserted. Every one of the series of propositions which are believed to be connected by these implications can then be believed, by themselves, to be true.

Inference is, in short, a form of judgment and, like all judgment, it takes the leap of belief; it introduces a new attitude of mind toward propositions. But it differs from simple judgment in that it must take two leaps of belief; it must accept the truth of an implication (or series of implications) and of the premise of this implication (or series of implications). Thus, from surveying propositions and asserting that they are connected in certain ways, it passes to the affirmation of these propositions apart from their connections.

That a belief in the truth of a proposition implied by a true premise is justified follows from the nature of implication. Anything implied by a true premise is true; for the truth of the implication, "p implies q," does not admit the possibility of p being true and q false. To believe p, and to believe that p implies certain propositions, is inconsistent with believing that these latter propositions are false. And though one is not *compelled* to believe the consequences of p when he believes p (for he always has the alternative of being sceptical), he cannot consistently believe that these consequences are false. Here as always the goal of his belief is consistency.

The two principles which underlie inference, then, are: (1) that any true proposition is true apart from its connections with other propositions, and can be asserted on its own account; and (2) that a true implication and a true premise always yield a true conclusion. These principles permit us to move in thought from one assertion or belief to another; to wipe the slate clean and start afresh with each new conclusion.

When a formal deductive system is taken to refer to certain objects, it is asserted, and if its postulates and rules of substitution are true for these objects, the conditions necessary for inference are satisfied. But it is more important still to observe another fact which follows from the complete interpretation of a deductive system. *The conditions of truth and of significance in the system become the same.* Every proposition which has a meaning is true, the system contains no significant propositions which are false.

The symbolic manipulations, which are the inferences of the asserted system, arise in the unasserted (purely hypothetical) system from rules of syntax which prescribe how the symbols are to be taken together as significant groups — rules analogous to rules of grammar. General principles of symbolism, a "universal grammar," necessary in all systems or "languages," are laid down, and within this framework special conditions of significance are provided for, so that each deductive system becomes a special "language." Its postulates state nothing more than what expressions are to be considered significant, what symbols are identical in meaning, and what symbols can be substituted for one another without rendering the expressions in which they appear meaningless. The system differs from more concrete languages in that its "words" and "phrases" refer to no objects but are variables, grouped about certain fixed elements, *viz.*, certain logical forms. These logical forms are the sole subject-matter of the uninterpreted language; they are em-

bodied in the "words" and "phrases," the symbols and groups of symbols, but they are not asserted to exist elsewhere. The language asserts no truths but simply surveys logical forms. On the other hand, when the system is completely interpreted, the language that previously asserted no truths becomes a language that asserts nothing but truths; there is no expression in it which is both significant and false.

The so-called "false propositions" of a completely interpreted deductive system are "pseudo-propositions," that is, meaningless collections of the symbols, and not significant groups permitted by the syntactical rules. Such "pseudo-propositions" can be introduced only by violating the conditions of significance. In the postulates for serial order,[1] for example, the group $(a(Oa))$ does not enter; it is nonsense so far as the syntactical plan of this "language" is concerned. Interpreted as the system is interpreted, this pseudo-proposition asserts that "any point, a, precedes itself," but though this idea can be expressed in words, it cannot be significantly stated in the symbols of the system. Thus it is superfluous (in fact absurd) to say that $(a(Oa))$ is false, for only a significant expression can be false.

Completely interpreted deductive systems are therefore *completely inferential*. Every transformation which conforms to the "grammar" of the system is an inference; every "grammatical" expression is true.

IX

Ordinary language, on the contrary (as well as the system of mental images), is *incompletely inferential*. The conditions of significance and truth for its expressions are not the same. Clearly if one follows merely its grammatical rules, he will not be led from true premises to true conclusions. Only some of the manipulations of its words and phrases will be inferences. But the process of thought, as it is carried on in language and in images,

[1] See above, sec. vii, note.

presents an interesting analogy to the transformations of formal deductive systems, for thought in these former media is also the manipulation of symbols.

Thought has two aspects: it is both dynamic and static; and inference belongs to its dynamic side. Inference is a movement of thought coupled with belief, but strict inference arises in a wider setting of thought-movement — a setting which Hobbes calls "mental discourse" or the "train of imaginations."

"Mental discourse," beginning with presented objects, mirrors a world of fact in a world of representations; it builds simple symbols which refer to objects into complexes, which may or may not correspond to objects. The drama of "ideas" makes use of symbols as its puppets; the symbols combine and recombine, the puppets move on the stage, and one grouping succeeds another to the end of the piece. Words and images are the primary instruments of this "mental discourse." They combine, dissolve, and recombine, pass and repass in new and changing unities, and here and there they precipitate a reference to fact. Such discourse is the free play of the imagination, musing or dreaming, rather than deductive reasoning, but it is akin to deductive reasoning in that it is a process of substitution within a plan of syntax. Images (both simple and complex), words, and phrases, take the places of one another in groups; the group that was now of one form assumes a different form; but throughout all these mutations, the plan of syntax on which the system of images or of words is based is never violated. The "mental discourse" confines itself within the limits of intelligibility which this plan imposes. Yet this discourse is not inference, as it would be in a completely interpreted deductive system, for the plan of syntax within which it is carried forward cannot be so interpreted that every significant proposition to which it gives rise is true. There is no difficulty in abandoning fact for fancy within

the systems of language and images; but in a completely inferential system nothing that is conceivable is false.

We cease to dream and begin to infer when our purpose in the manipulation of symbols is to arrive at truth. A completely inferential (and interpreted) deductive system is so constructed that this purpose can be accomplished merely by following its rules of significance, for these rules sum up a multitude of implications which hold for the subject-matter. But the syntactical plans of language and the imagination do not run parallel to logical connections of the subject-matter. In these media, only a limited number of expressions will imply or be implied by others, and so can be inferred from others. These expressions will form a solid island of true inferences in a sea of fancies. Yet there is a marked analogy between free imagination, "mental discourse," and formal deductive inference. The constructive imagination is a genus of which the process of formal deduction is a species; imagination is necessary to reason.

X

The principal points that have been brought forward in this examination of formal deduction are the following:

For formal logic, all that is essential in any subject-matter is its logical structure, and this structure can be isolated and studied in itself through uninterpreted symbols whose forms and their connections alone are considered. Such uninterpreted systems of symbols are not without meaning, however. In the context of logical structure, all the elements of these systems, with the exception of their logical forms, are variables, that is, symbols of undetermined meaning; and the groups as wholes have significance, though they refer to no objects, for any symbolic group with a structure is significant. The forms to be studied are *presented* in the very symbols through which they are studied.

The postulates of these systems are plans of syntax which de-

termine what groupings of the symbols are permissible, that is, what the possibilities for thought in the system are; and the whole becomes a special "language" with a "grammar" of its own marking the boundary between sense and nonsense for the system. All special plans of syntax assume the general principles of symbolism: (1) the principle of group significance; (2) the distinction between symbols of unity and terms; (3) the law of identity and (4) the law of contradiction. From these principles, it follows that the system must be composed of groups of terms joined by symbols of unity; the same symbols must always be used with the same significance, though symbols of different characters and forms can be taken as identical in meaning; and symbols that are originally distinct in meaning must always be distinct.

All the significant complexes of the system are derived from certain basic complexes by substitutions, and this process of transformation by substitution *is* formal deduction; it is the manipulation of symbols within a plan of syntax. Two general rules of substitution are always followed: (1) the same symbol can be substituted for the same symbol, and symbols of identical meaning can be substituted for one another, in any complex; (2) all substitutions must be carried through completely. In addition to these general rules, every system has its special rules of manipulation, and among these may appear certain "equations of structure," definitions, and specially postulated implications. The latter permit the substitution of one group for another only when this other group appears as a conclusion of a series of deductions. Deduction in these systems is nothing but the manipulation of the symbols according to these rules, as is illustrated by the Boolean Algebra, which, though it can be completely interpreted in terms of classes and class relations and operations, can be stated as a system of uninterpreted groups and rules of substitution.

A rule of substitution, in general, is based on a relation between the functional ranges of the variables. If symbols coincide in their ranges of functional variability, they can be mutually substituted for one another; and if the range of one includes that of another, the former can be substituted for the latter, for it will be a functional value of the latter. With respect to variables of a less inclusive range, those of wider range are "functional constants"; while the variables of narrowest functional range in any system — those for which all other expressions of the system can be substituted, but which can not be substituted for these expressions — are the most general "functional variables" of the system. Both functional constants and functional variables are undetermined in meaning; both are "interpretational" variables.

If a group is a possible value of the variable terms which enter in this group itself, this group is *operational* rather than relational. Operational are distinguished from relational complexes by the use which can be made of them in formal deduction, by the fact that they can be substituted in themselves for their own terms. Thus they give rise to an infinity of distinct possible forms.

When a deductive system is interpreted, the arbitrary manipulations of the symbols become inferences, for the postulates and rules of substitution then embody implications, and if the postulates are true, the theorems are also true. To perform an inference is to pass from a premise that is believed, by way of an implication that is believed, to a conclusion that is believed on the strength of these other beliefs. Inference cannot be separated from belief and assertion; it cannot be purely hypothetical and the principles on which it rests are: (1) that any true proposition is true, and can be asserted, apart from its premises; and (2) that what is implied by a true proposition is true. Inference is "the dropping of a true premise."

In a completely interpreted deductive system, whatever is significant is true; any of the permitted transformations of meaning lead from true premises to true conclusions, so that these systems are *completely inferential.* They are to be contrasted with *incompletely inferential* systems, such as language and the imagination, in which there is a discrepancy between significance or conceivability and truth. Though these latter systems resemble the former in that thought is carried forward in them by the manipulation of symbols within a plan of syntax, they differ from the former in that the pursuit of their rules of significance, alone, gives rise to fancy and free imagination rather than to inference.

These formal deductive systems, in which everything but the logical structure of the matter referred to (if there is a matter referred to) is pruned away, show much more clearly than other symbolic systems how symbols copy the forms of facts and why the logical structure, which is present in the symbols as it is in the objects, is the essential bond of meaning between the mind and its objects. The mirror of thought, which reflects in the beginning a world of concrete facts, may send back a reflection of forms alone and these forms may belong to no world of fact, they may be merely possibilities for thought embodied in symbols. But the structure of these possible concepts will determine what they *can* stand for; if they represent objects, these objects must correspond to them in logical form. Whether formal deductive systems are interpreted or uninterpreted, thought reaches in them its maximum of clarity.[1]

[1] The views of formal deduction and truth here set forth are closely similar to those of Leibniz as interpreted by L. Couturat in *La Logique de Leibniz* (Paris, 1901). "In an unpublished fragment relating to the universal language," says Couturat (pp. 88, 89), "Leibniz imposes another condition on signs: it must be possible to deduce from their form alone, and from their composition, all the properties of the concepts which they represent. . . . Their combinations must depict for the imagination the logical connections of the corresponding concepts, so that the composition of the signs agrees with the

composition of the ideas, following an exact analogy. . . . Moreover, not only does the symbolism translate the thought under an intuitive form, but it serves also to guide, to relieve, and even to supplant or replace thought. Just as the combinations of the ideas are represented by combinations of the corresponding signs, so the operations of the mind — that is to say, the reasoning which is carried on with these ideas — is expressed in concrete and sensible operations performed on the symbols. The abstract laws of logic are thus translated into intuitive laws which govern the manipulations of signs."

To the nominalism of Hobbes "Leibniz replies peremptorily," continues Couturat (pp. 104, 105), "that, if signs are arbitrary, the relations between signs, which express or constitute propositions, are in no sense arbitrary; and that they (signs) are true or false according as they correspond or not with the relations of the things signified. Thus truth consists in the connection of signs so that they agree with a real and necessary connection of ideas or objects, which does not depend on us; or, in other words, it consists in that similarity of the relations of signs and the relations of things which constitutes an *analogy* in the proper mathematical sense of the term, that is to say, a proportion or equality of relationships. . . . The choice of signs and the definitions of words can, then, be arbitrary, but the linkage of the words and signs does not become so; and it is in this linkage alone that truth and falsity reside. We can even change the system of signs at will without changing a truth or making it dependent on our wishes; for, whatever symbols are chosen, there will be an arrangement of these symbols — and only one arrangement — which will be true, that is, which will correspond to the real order of things or facts. There is then an *analogy*, not only between signs and objects, but between different systems of signs in so far as they express the same reality.

"This necessary rather than arbitrary order which exists in things is, though unknown, the objective basis of all truth. Once a certain system of arbitrary signs or a certain set of conventional definitions has been adopted, it no longer depends on us what combinations are true and what false; and this proves that truth, although it resides solely in our minds, rests on a principle outside us and expresses symbolically a reality of some sort."

CHAPTER VIII

THE METAPHYSICS OF KNOWLEDGE

I

A THEORY of knowledge must come at last to metaphysics, for the aim of knowledge is to grasp reality, and without some notion of what reality in the broadest sense of the term is, we cannot say whether knowledge succeeds or fails. The limited reality of which we have spoken up to this point — which can be given in or inferred from an experience consistent with the whole of knowledge — is not enough. It is always possible that what can be thus given or inferred is merely an appearance, and that reality is something deeper, inscrutable to perception and perhaps even to reason. And so we must set out without any artificial restriction of our concept of reality to answer, or rather to sketch the barest outlines of answers, to these fundamental questions: Can reality be known? If so, what is reality and how is it known?

The first question is prompted by the sceptical impulse of knowledge to self-criticism; it arises from a mood of doubt which is unfamiliar to very few men. Walt Whitman speaks of this mood as "the terrible doubt of appearances,

> Of the uncertainty after all that we may be deluded;
> May-be the things I perceive — the animals, plants, men, hills, shining and flowing waters,
> The skies of day and night — colors, densities, forms —
> May-be these are (as doubtless they are) only apparitions, and the real something has yet to be known.[1]

[1] Walt Whitman, "Of the Terrible Doubt of Appearances," first published in *Leaves of Grass*, 1860.

Most of us easily shed this "terrible doubt of appearances." Practical life obscures it, common sense engulfs it, and only poets, mystics, and metaphysicians are left to worry over it. But those whom it haunts find that there is no compromising with it; either all knowledge gives way before it or else thought is driven back and back till it touches first principles — and lays the foundations of a system of metaphysics.

The peculiar thing about this question is that it cannot be significantly answered in the negative. Either we are condemned to perpetual asking without an answer or else we know reality. The sceptical impulse of knowledge to self-criticism defeats itself, attains no end, unless there is knowledge, not merely knowledge of appearances but of reality. A criticism of knowledge makes use of the very thing it criticizes, and unless the argument is to become circular or lead back indefinitely from premise to premise, the knowledge on which the criticism is based must itself be exempt from criticism.[1] If knowledge fails (in some sense) to grasp reality, in knowing this fact it attains an ultimate truth and does not fail completely; and if it succeeds, the knowledge of why it succeeds cannot stand in need of justification, and so on through a series of justifications that has no end.

This suggests either that no final criticism of knowledge is possible, that suspension of judgment is the way of sanity (and this means equally that knowledge cannot be vindicated or dis-

[1] This idea is clearly put by Aristotle; see W. D. Ross, *Aristotle* (1924), p. 45. "There are, says Aristotle, two errors (with respect to the ultimate validity of knowledge) which rest on a common basis. There is that of supposing that knowledge implies either an infinite regress from premise to premise in order that nothing may be accepted as unproved, or else the acceptance of unproved and therefore unknown premises, and that knowledge is therefore impossible. And there is the error of supposing that knowledge is possible but proceeds in a circle — truth being thus reduced to the mutual implications of propositions none of which are independently known to be true. The common basis of the two errors is the assumption that proof is the only way of knowledge, and against them both he affirms his principle that there are first premises which neither need nor admit of proof."

credited); or that there are first principles, final insights into the nature of the real, on which all knowledge rests. The former alternative is scepticism; the latter, the necessary starting-point of any metaphysics.

And yet there have been attempts to compromise with doubt by answering this initial question — can reality be known? — in the negative, attempts to maintain that knowledge is of appearances only and that reality is hidden and unknowable. Herbert Spencer makes his bow in the opening chapters of his *First Principles* to the eternal mysteries of the universe, leaving this empty but venerated realm of the Unknowable to religion and metaphysics, and choosing for science the full and certain area of the Knowable. A strange, unstable dichotomy! There is to be no poaching of metaphysics and religion on science, and none of science on religion and metaphysics. Each goes its own way, metaphysics to the Unknowable, which is the real, and science to the Knowable, which is the unreal or half-real. Metaphysics becomes an unfulfilled aspiration toward the ultimate, and science a description of phenomena which, so far as we can tell, are less than shadows flitting before the senses.

The agnostic compromise with doubt attains no better success than this. Clearly it is no compromise. Agnosticism is not a half-way house on the road between scepticism and metaphysics. There is no half-way house. If one accepts any truth, he accepts a standard by which he discriminates the true from the false, and beneath this standard is found his metaphysics. From the beginning of thought a concept of reality is present, making itself clearer as thought goes forward but never abandoned — unless thought thins out into scepticism.

Men are for the most part neither self-conscious metaphysicians nor self-conscious sceptics; the temper of most minds strangely blends these two tendencies. But if either of these incompatible strains is brought to light, it must exclude the other.

The scepticism which is fully aware of itself witholds all belief, ventures on no single affirmation, — not so much as the affirmation that belief is fruitless and unjustified, — and is thus reduced to silence, or at best to a vagrant roaming among possibilities, none of which is affirmed to be actual. But if one asks himself how it is that he can assert any truth, even the most meagre, he begins to uncover a metaphysics, for truth is never anything less than knowledge of the real. To become alive to any truth he must bring to the surface of thought the concept of reality he now discovers to be buried there.

The description of knowledge which has occupied us so far takes its departure from the point of view of the plain man and hence is neither self-consciously sceptical nor self-consciously metaphysical. It appeals to *experience* in a wide meaning of the term, witholds judgment on ultimate questions, but at the same time is aware of its assumptions and does not assert that they are more than assumptions. The upshot of the positive theory of knowledge is this: the *truth* of ordinary experience is the correspondence of concepts through their structure with real objects, and this reality can be given in or inferred from an experience consistent with the whole of knowledge. Whether reality as such has a structure, whether the objects that appear in experience are finally real, whether the mind in knowing is separated from or joined to an ultimate reality — all these questions remain untouched. And yet, if they are swept aside, the door is left open for scepticism. In the face of these questions the limited truth and reality of the positive theory of knowledge must expand into a final truth and a full reality or contract into a flux of impressions and ideas — leaving only doubt. The positive theory of knowledge, including as it does the two incompatible strains, is unstable; it is a metaphysics tinctured with scepticism, a scepticism tinged with metaphysics. The task of speculative philosophy is to bring to complete self-

consciousness the one or the other of these warring attitudes of mind.

It is this aim which distinguishes speculative philosophy from the special sciences. Science tolerates the frame of mind that joins scepticism to the assertion of truths. The scientist does not doubt that he is approaching a completer knowledge of reality, though he could not say why this is so without becoming a metaphysician; and yet he refuses all commerce with ultimates and thinks of his premises as working assumptions which he is not averse to discarding for better ones. Hesitating to affirm that he has reached a final truth, the scientist still believes that his growing knowledge is an indefinite approximation to such a truth. Here the metaphysician makes his point. If one's standards of truth are not themselves finally true, if one does not in some way already grasp reality when he sets forth on the journey of thought, he cannot even conceive of attaining truth.

The method of science is the method of assumption, hypothesis, postulation, and this way of thought falls between scepticism and metaphysics; it holds the two in an unsteady balance within itself. A full-grown metaphysics cannot rely on postulates or assumptions, for if these assumptions are false the whole of knowledge collapses, and *assuming* them to be true does not make them true. A full-grown scepticism, on the contrary, is equally receptive to all assumptions, not excluding contradictory ones.

II

A tradition of critical philosophy which purports to be neither sceptical nor metaphysical, nor yet to be poised as are the special sciences between these extremes, appears in the thought of the eighteenth and late seventeenth centuries. Kant is the father and Locke the forerunner of this more carefully reasoned agnosticism. By an examination of the nature of knowledge, this critical philosophy hopes to determine whether or not meta-

physics is possible, and it concludes that metaphysics is impossible — that knowledge, being limited not only in extent but in its very essence, cannot penetrate through appearances to reality.

The distinction between appearance and reality is as old as thought; indeed philosophy was bred in the suspicion that the real world is not what it appears to be, and the first philosophers set out with the confidence of adventure to discover the ultimate beneath the surfaces of things. But a second suspicion followed speedily on the first: that knowledge is without power to reach this ultimate. And so thought turned inward, its confidence shaken, to a criticism of its own capacities and aims.

Whether knowledge is able impartially, and without begging the question by assuming its own validity, to criticize itself, is a query that does not occur to the agnostic. Pretending with Kant that this self-examination of thought will show whether a final truth can be reached, the agnostic forgets that his own theory is built on a metaphysical first principle, and that he himself claims one final insight — namely, that knowledge is confined to appearances. Though all other truths are in his opinion limited and phenomenal, this truth — that we cannot know the ultimate — is absolute.

Stated thus in outline, critical agnosticism appears self-contradictory. It is as if we were saying that by sight man can discover that he cannot see. Yet both Locke's *Essay Concerning Human Understanding* and Kant's *Critique of Pure Reason*, having begun with the conviction that knowledge by internal criticism can reveal its own frailties, arrive at this conclusion. Kant and Locke have fostered a tradition which gives epistemology a peculiar authority in the field of philosophy, but their epistemology is in reality a metaphysics. The philosophy that Kant terms *critique* is speculation of a most insidious sort, disguising itself as it does under another name. Though its aim is to discover "whether such a thing as metaphysics be at all possible,"

it is itself impossible without the metaphysics whose wings it hopes to clip. The epistemological tradition of Locke and Kant is an unsuccessful attempt to straddle the issue between scepticism and metaphysics.

Before the Kantian critical philosophy was born, a reply to it was framed by one of the metaphysicians whose dogmatism Kant condemned. In Spinoza's *Tractatus de Intellectus Emendatione*, Kantianism is forestalled by the following argument: "In order to find the best method of investigating what is true, we must not stand in need of another method to investigate this method of investigating, nor in need of a third one to investigate the second, and so on to infinity. For by such a method we can never arrive at a knowledge of what is true, nor at any knowledge whatever. For it is the same thing as with artificial instruments, of which we might argue in the same manner. For in order to work iron a hammer is needed, and in order to have a hammer it must be made, for which another hammer and other instruments are needed, and so on to infinity; and in this manner any one might vainly endeavor to prove that men have no power of working iron. But in the same way as men in the beginning were able with great labor and imperfection to make the most simple things from instruments already supplied by nature . . . so also the understanding by its native strength (*vis sua nativa*) makes for itself its intellectual instruments wherewith it acquires new strength for other intellectual works, and so gradually proceeds until it attains the summit of wisdom." [1] This native strength of the intellect is its power of insight into reality, which is present in thought from the first.

The severance of appearance from reality, on which most criticisms of knowledge are based, may be more or less complete. It may be sharp and irreparable as in Kant, or it may be partial and reconcilable as in the Absolute Idealists, who be-

[1] B. de Spinoza, *Tractatus de Intellectus Emendatione*, sec. 6.

lieve that "there is no truth but the whole truth," and yet that no appearance is denied its small grain of reality and truth. The breach between appearance and reality once opened must be healed, and the reason is not far to seek. What appears must have some sort of reality or it could not appear; it would be simply nothing. The important question then is, not how are appearance and reality separated, but how are they united? — how, in Spinoza's language, does reality manifest itself in attributes and modes? To hold that there is a world of absolute appearance cut off from a world of absolute reality, as Plato's realm of opinion is from his realm of ideas [1] and Kant's noumenal realm from the phenomenal, is to fall into a difficult dilemma. Either the world of absolute appearance or the world of absolute reality should be dropped; the two refuse to stand side by side unrelated. Just as Aristotle attempted to bring the Platonic ideas into the concrete matter of experience, so the Post-Kantians attempted to raise the Kantian phenomena toward the level of the noumena by making the phenomena finite parts of an infinite and absolute whole.

III

Before we consider ways of healing the breach between appearance and reality, let us examine the breach itself in its most extreme form, as it occurs in Locke and Kant. It arises here from a theory of mind-isolation which places appearances in the knowing subject and leaves reality out in the cold beyond the grasp of the knower.

Locke's metaphysics of mind-isolation rests on two premises: that "all our knowledge is conversant only with ideas," and that ideas are of qualities only, never of substances, which are the substrata of qualities. Locke's view presupposes the passiv-

[1] There is a question as to whether Plato intends completely to sever his two worlds.

ity rather than the activity of mind; a static screen of ideas imposes itself from outside between the knower and reality, never to be swept aside.

This is the crudest and simplest statement of the mind-isolation theory — the theory of *epistemological dualism*. Since Berkeley, it has had little currency in this form. The crux of Locke's argument is the second premise, the distinction between substance and its qualities; but just why substance, which Locke sometimes says is known in a confused way and other times not known at all, should be retained — why qualities (appearances) do not make up a self-sufficient and complete reality — does not become evident. Locke's substance is nothing more than an arbitrary limit set to knowledge for no other reason than to satisfy the common-sense prejudice that things must have a substratum in which their qualities inhere, and the slender thread that holds together substance and its qualities snaps under the tension of Berkeley's criticism; qualities alone are left, the unknowable x, substance, being cancelled out.

Locke's first premise, that "knowledge is conversant only with ideas," supports the theory of mind-isolation if one thinks of ideas as essentially disparate from real objects and (in Locke's language) real qualities. One can for example suppose that real objects cause ideas different from themselves. How one can know this is another question, for apparently he must go beyond ideas to do so. Therefore, if knowledge is conversant only with ideas, it is by hypothesis not conversant with objects which cause, bound, or limit ideas; and so the statement that knowledge is *restricted* to ideas has no foundation, unless it affirms merely that "we know what we know." This is again the typical difficulty of agnosticism: if the mind is isolated, it would need to escape its isolation to know this fact.

Though Kant's formulation of the theory is more profound

than Locke's, it comes in the end to the same thing — agnosticism and mind-isolation. Kant does not argue from the distinction of substance and its qualities, and he supports the notion that knowledge is conversant only with ideal objects by negative if not by positive proofs. Kant insists as against Locke that the mind is active in cognition; this is perhaps his most important observation on knowledge as a whole. Yet from this premise he draws the amazing conclusion that in the very act of knowing the mind shuts out from itself the reality (the thing-in-itself), and thus becomes ignorant of what it is striving finally to reach. As soon as the work of knowledge begins, the mind like a snail in its shell withdraws within itself. The concepts (categories and forms of perception) which determine the mind's essential directions of activity, far from causing it to expand outward to a fuller apprehension of the real, which it has somehow touched in "the original receptivity of the senses," turn it away toward ideal objects which correspond in no intelligible way with the ultimate reality. Knowledge is not a passive reception of impressions (Kant argues), but an active assimilation of them under concepts,—the mind is not a blank sheet, but a working organic whole with a structure, — and so knowledge is confined to appearances, which are the products of the understanding at work with concepts on the materials of the senses. Thus the objects of knowledge form a neat and orderly closed system through which the wind of reality does not blow. The safe and sheltered island mapped out by thought is untroubled by the waves of the *ding-an-sich* which beat along its shore.

Why must this strange introversion go with the notion that the mind is active in knowledge? Granted that thought works with concepts and categories, that the mind is not like a vat of dough, merely receptive to external impressions, but that it does something in cognizing an object. Still, it does not follow that this activity is a contraction of the mind within itself. The con-

cepts along the grooves of which cognition is directed may lead the mind outward to a more complete knowledge of the real, rather than twist it inward upon itself. The more reasonable corollary to the observation that the mind is active in cognition is the exact opposite of mind-isolation. The activity of thought through concepts, intentions, (that is, symbols with their attendant mental attitudes), should bring the mind into more perfect and stable relations with the reality it touches in "the original receptivity of the senses." We cannot conclude that the organized knowledge of perception and experience is not knowledge of realities simply because thought plays a part in the organization. If reality itself is a complex whole, with "a time-less subtlety of complexity," [1] surely it is not strange that reality should be clearly apprehended only by an organized and active mentality. What is needed in place of the Kantian theory of isolation is the notion that mind by its activity joins itself to real things in knowledge, that is, an *epistemological monism* which takes account of thought-activity in cognition. Mind grows into a cognitive unity with the reality it originally knows only in fleeting and momentary glimpses; *the mind in knowing is actively continuous with real objects.* We shall presently say more of this continuity.

Kant's tidy world of experience, created by the mind at work with concepts and categories, could be peopled only by objects for which these concepts and categories are universally and necessarily true; and so necessity is brought into connections of "matters of fact" (as against Hume) at the cost of severing the whole world of fact from a vast and incomprehensible realm beyond fact. One feels cheated, tricked, by this vindication of *a priori* knowledge. It is a peace without victory. And what of this theory of mind-creation itself? Is it, too, nothing but a mind-creation? If not, Kant is thrown back on the insight into real-

[1] A. N. Whitehead, *The Principles of Natural Knowledge* (1919), p. 15.

ity of which Aristotle and Spinoza speak. Kantianism is slain by its own hand unless its own first principle — that knowledge cannot go beyond appearances — is apprehended as not itself an appearance, apprehended by a kind of reason not subject to the limiting categories of ordinary thought. But to admit such a rational insight or "intuition" would have been a return to dogmatism of the sort Kant hoped forever, by a dogmatism peculiar to himself, to demolish.

Kant's "dialectical" arguments attempt to show the futility of offering rational answers to metaphysical questions. But the cure is more reason, closer analysis, clearer concepts, and not, as Kant would have us believe, the surrender of rational metaphysics. A growing knowledge is bound to meet contradictions and thus to alter its concepts, and here lies the correction for the Kantian antinomies as well as for the more numerous contradictions which Mr. F. H. Bradley — following the scent of the dialectical argument even further than Kant — finds, in his *Appearance and Reality*, to be inherent in discursive thought. This much truth can be distilled from both the Kantian and the Bradleyan dialectic: it is fatal to employ the concepts of the special sciences for unscientific, that is to say, for metaphysical purposes. Metaphysical categories (if there are any such) must be the widest possible categories. Mr. Bradley finds that thought in the effort to complete itself does not attain to wider concepts, but "commits suicide" by transcending its own forms, thus making way for the dim apprehension of an Absolute of which discursive knowledge gives only an imperfect and restless foresight. In this way Mr. Bradley escapes agnosticism to fall into something very close to mysticism. Kant's conclusion from his antinomies is, on the other hand, the most glaring and insoluble antinomy of all. An inherently defective reason becomes able to know its own defects; a mind isolated from reality and confined to a world of phenomena becomes negatively but not positively

aware of its isolation. Even so the mind craves a positive knowledge of this reality and its relation to the world of experience. The noumena do not cause phenomena, for causation is a connection between objects of experience only. Indeed, the conclusion ought to be that the noumena are not related to the phenomena, for *relations* hold only between phenomena. The thing-in-itself, the ultimate reality thus severed from its appearances, is an entity which plays the same part in knowledge as *zero* in arithmetic; the addition or subtraction of it does not alter the value of the other elements. The outcome of the Kantian "dialectic" is that knowledge of that which limits knowledge is negative only; which amounts to saying that knowledge is limited in a way that is not known by something that is not known. And it is a short step to the conclusion that knowledge is not limited, and hence to a rejection of the whole theory of mind-isolation.

One other conclusion stands out in the "dialectic." Reason is always attempting to complete the synthesis of experience, to effect a "complete unity of knowledge . . . by which that knowledge becomes not only a mere aggregate, but a system connected according to necessary laws." Kant sees reason as dynamic and directed toward a goal, which it approaches but does not reach. Not content with a slow process of growth and correction, it takes disastrous short-cuts, falling thus into self-contradiction. But this incompleteness of knowledge is not a sufficient ground for the agnostic separation of appearance from reality. It is one thing to assert that knowledge at any particular time does not give us the whole of reality. This is a mild confession of ignorance and finitude, a gentle and tolerable agnosticism, which is the part of intellectual humility in the face of changing concepts, shattered theories, and discredited beliefs. It is quite another thing to assert that knowledge never grasps more than an appearance. A knowledge that is not complete in

detail must nevertheless reach through to reality; otherwise, it is not knowledge.

IV

Four major points emerge from this discussion of agnosticism. (1) Any final criticism of the validity of knowledge must rest on metaphysical premises. The epistemological tradition which pretends to determine the limits and extent of human knowledge — especially, whether metaphysical knowledge is possible — appeals itself to metaphysical premises which are true without limitation. Appearance and reality cannot be held apart. (2) Though the mind is active in cognition, this fact need not lead to the conclusion that the mind shuts itself out from reality, building a world wholly its own. To the theory of mind-isolation must be opposed that of the continuity of the mind with real objects in the act of knowledge. The activity of the mind in the channels of concepts brings it into more complete touch with the reality of which sensation gives only passing glimpses. Through the use of concepts knowledge expands outward to reality, rather than inward to its own ideal world. (3) At every moment knowledge is incomplete, the whole of reality is not known in detail; the truths of to-day may be reversed by those of to-morrow. Yet the very notion that knowledge is incomplete demands that some knowledge be ultimate. Thus (4) a rational insight or intuition is needed to take account of metaphysical truth, for the truths that vindicate (or enable us to repudiate) all others cannot themselves be vindicated or repudiated, nor can they be merely postulated. They must be apprehended as final.

These conclusions must be expanded and correlated with the description of knowledge previously given.

V

The essence of the first point is this: if there is any knowledge, there is knowledge of reality. An appearance is never a bare appearance, for if it is true that it is an appearance, then it has some status in reality. It is *really* an appearance. The opposition between appearance and reality becomes less and less significant the more one contemplates it. The question is, not whether what appears is a reality, but what sort of reality it is. What is its place in the scheme of reality? An appearance deceives when it is misplaced in reality or judged to be all there is, yet even a deceptive appearance is something; it cannot be dismissed as not being. To hold that appearances have no reality is to deny that they are appearances, and hence to wipe them, and all the problems connected with them, clean off the slate. If the distinction between appearance and reality is to have a meaning it must fall within experience, and not between what is known and what is unknown or unknowable. It must fall within an apprehended reality. The very terms "appearance" and "reality" mislead one. We ought not to say that the reality we know differentiates itself into appearances, but into parts, aspects, elements, modes, which never lose their essential reality. What we call "appearances" are fragments of reality, and *reality* is in the fragments no less than in the complex structures, that is, objects, situations, facts, into which these fragments fit to form a world.

The task of metaphysics is to describe what is most general (and hence least noticeable) in this apprehended reality, however fragmentary the part apprehended may be. Metaphysics cannot look for the real beyond or behind experience, nor in an ideally complete experience. It must, like any other effort to think clearly, dig into the basic units of knowledge which are presentations or wholes of experience. Metaphysics is the most searching possible analysis of these wholes.

This analysis should reveal what part the mind plays in apprehending these wholes. Does the mind in some far-reaching sense create their reality? Or do they in some mysterious way enter the mind from outside and mold it to their own forms?

VI

There has always been a tendency to break up the act of knowledge into sensation and thought, sensation being passive and thought active, and this has led to curious results, some of which have already been observed. In the main, those who have stressed sensation and the passive rôle of mind have insisted that reality is external and indifferent to mind, and that reality becomes known through impressing itself on the mind; while those who have stressed thought have made the opposite claim — that reality is mental, rational, a thought-creation, and that it is known through the mind's activity. There is something to be said for both of these views for the reason that the act of knowing is not sensation with thought added, or thought with sensation entering accidentally in it. To know is to be at once active and passive, to receive an impression and reflect upon it. The act is a single whole in which thought and sensation are blended; what is impressed on the mind is assimilated to, and not distinguished from, what is meant, imagined, thought. Concrete knowing is the active assimilation of reality through *sensational-thought*. We have spoken of this before as *presentational-thinking*. It is impossible to say where the receptivity of the mind leaves off and its activity begins. Certainly it is highly artificial to draw this line at sensation, for there is no such thing as a pure sensation in abstraction from the cognitive act as a whole. As an element within this whole, sensation or sensing no less than thought is active. Moreover thought, as an element within the cognitive whole, has its passive side; it comes sharply against the thing it means given in experience, and

this encounter checks and at the same time fulfills its intentions.

What has been previously said of thought must be recalled: that thought is essentially reference through symbols, an outward reach of the mind toward objects. There is a distinction between pure thought, which is reference and nothing more, and perceptual thought — which is reference when what is referred to is also immediately presented. The latter is what we are now speaking of as "concrete knowing." This is the basic form of knowledge. Pure thought, purely mediate knowledge, is knowing in which the element of presentation (that is, passivity) is refined away, so that the thing meant is no longer given but merely intended. I do not know the other side of the moon concretely, but I do know concretely the façade of the house opposite upon which my window looks. Since pure thought is a distillation, an abstraction, from concrete thought, it is to the latter that we must look for the fullest relation of the mind to real objects in knowledge.[1]

The view that objects impress themselves on the mind through the senses is insufficient for many reasons. Sensations are nothing in themselves; they are always parts of larger wholes. It is not correct to say that we "sense" relations, facts, complex structures; yet we experience these things. Cognition is always recognition; memory plays a part in it, and what is given has a reference before and beyond itself. There is an adjustment to experienced objects; they are classed, catalogued, fitted into a scheme of things in the very act of perception. The fact that objects are experienced as "black," "white," "round," or even as "objects" or "sense data," shows that a concept is actively

[1] This distinction between pure and concrete thought corresponds to Aristotle's distinction between potential and actual knowledge. In actual knowledge we grasp the real; potential knowledge needs something, a fulfillment or realization, to become actual knowledge. See W. D. Ross, *Aristotle* (1924), p. 171; also Aristotle, *Metaphysics*, Bk. XIII (M), ch. 10.

at work in the apprehension of them. Unless objects can be *consistently* assimilated to the knowledge already present to us, they puzzle us; we feel that they are not clearly apprehended, that we have not yet succeeded in perceiving them. Thus not only is a single concept present in the apprehension of objects, but a whole body of theory — certainly the whole body of common sense — presses forward to enter into the act of knowing, and the perception remains unstable until it is reconciled with this body of knowledge. All these and many other arguments point to the inadequacy of the notion that knowledge is a passive reception of impressions. Concrete knowing feeds on the whole of knowledge; what has been experienced, and the use the mind makes of it, determines what is and will be experienced. The direction of one's thoughts, meanings, intentions, coöperate in what he sees and hears, no less than does the external situation in which he happens to find himself. If the mind were a *tabula rasa*, there is no reason why first impressions should not be as clear, as complete and final, as last impressions.

Yet there is a stubbornness, a resistance, an alien character, in the objects of concrete experience which prevents us from taking the opposite point of view, namely that they are thought-creations, things merely meant and in no sense given. We do not invent the things we see about us; they force themselves in upon us, and though the mind goes forth through its intentions to meet reality, still thought comes in the end to something unyielding. If experience deceives us, we do not get rid of the experience by apprehending it under proper categories and thus doing away with the deception. The data remain, however we conceptualize them. It is impossible to convince one's self that the world is merely a coherent set of meanings. Whatever the activity of thought does in contributing to concrete knowing, there always remains a *matter* not its own creation. However thought may elaborate this matter, the matter persists, to con-

fine and complete concrete thought.[1] Of the infinity of worlds possible to purely abstract thought, one is realized, and it is this one which experience reveals to us. For abstract thought alone any other world might equally well have been.

The idealist, who looks on reality as a rational system of meanings or concepts, still finds it necessary to account for the stubborn givenness of this world; he cannot ignore the specific materials of knowledge; he must make it clear why *this* rather than *that* possibility is realized. And quite properly, not wishing to fall into the Kantian predicament, which places reality beyond the reach of thought and yet maintains that in some strange way this reality limits and molds experience, even though this experience is always of ideal (mental) objects only — not wishing to fall into this difficulty, the idealist distinguishes between subjective and objective thought, between relative and absolute knowledge. Finite experience is a fragment of the absolute experience, finite thought a part of the absolute thought. Through entering (partially) into this absolute thought, we know reality, and so the world about us is more than the creation of our own minds.

This idea is most simply put by Berkeley, who interpreted the whole of nature as a direct communication from God. In the hills, the trees, the stones, the events of history, we read the thoughts of the Deity; all are signs of what is passing in His mind. The stability of nature, the persistence and externality of perceptual objects, arise from the stability, persistence, and externality of God's thought. Illusion and error are man's dream; truth is the apprehension of what God compels us to know. Berkeley's idealism has a fresh, naïve beauty not found in his more sophisticated brethren, the Post-Kantians; but in essence it is absolutism.

Absolute Idealism, then, does not do away with the alien qual-

[1] *Matter* in the Aristotelian sense.

ity of the objects of experience, and at the same time it takes complete account of the work of thought in apprehending these objects. It does not ask us to believe that reality stamps itself on the mind from outside, that we are mere receptacles of impressions and ideas. Knowing becomes an entering into reality, a sympathetic understanding of a thought-world which is alien to and more inclusive than our own world of thought. The activity of the human mind is compelled and illuminated by the activity of the Absolute Mind in which it has its being.

VII

This is a heady doctrine, and one pauses to ask himself what it comes to. What is the Absolute Mind? Need one call it a "mind"? What significance remains in speaking of reality as essentially "thought"?

The chief argument for idealism, aside from the necessity of accounting for the mind's activity in knowledge, is one that has been mentioned above; the mind must be continuous with the reality it knows. Thus it is easy to leap to the conclusion that reality is mental, to Berkeley's *esse est percipi*. Yet objects refuse to be reduced to *my* mental states or *my* meanings; they persist, I believe, when my back is turned (and certainly Berkeley also believed this). The laws of nature, of mathematics and logic, are not *my* thoughts; I discover and do not invent them. And though Berkeley would have admitted that he "ate, drank, and was clothed with ideas," these were God's ideas and not his own. Reality must be sufficiently close to the mental to be known through the activity of my mind, but it must not be so completely mental as to be reduced to this activity. The idealist's argument comes to this: he is forced to distinguish "mental$_1$" from "mental$_2$", and what remains? Merely lip-service to a word. What he is saying, apart from this word "mental," is that his mind is so related to reality that it can know reality

through its activity. Obviously, no mind can be completely foreign to the reality it knows; the mind and its objects must be in the same universe; they must not be so wholly sundered that no relation, no sort of community or continuity, can exist between them. But it profits me nothing to insist that this reality is mental, a thing of thought. I mean that it is "mental$_2$" and not "mental$_1$" and these are very different.

"Reality" is the most inclusive of all possible terms; if any distinctions of meaning exist, these must arise within reality. There can be no significance in defining reality in one way rather than another, for whatever special term be taken as its definition, this term will have arisen as a distinct "somewhat" within reality, and it will lose its special meaning if it is made equivalent to reality. To speak of reality as mental is to spread mentality so thin that, to all intents, it becomes "not-mental." We are driven to qualifications — "objectively" and "subjectively" mental — to bring back the significance that has departed from the term.

One might as well recognize at once that reality is reducible to none of its aspects or modes, and be content with the term "reality" in its simplicity as undefined. For the logic of the situation will finally force him virtually, if not explicitly, into this position. The realist declares that "reality is independent of the knowing subject and not essentially mental, though it can be known"; the idealist — what appears to be the same thing — that "reality is mental (but not subjectively so) and dependent on mind (but not on the subjective mind)." Both agree on the fundamental point that the mind in knowing is continuous with real objects; both repudiate the Kantian dualism of idea and real object, of phenomenon and thing-in-itself. In this respect the American realists go further than many idealists; they assert not merely that the mind is continuous with the real objects it

knows, but that it is identical with them — or at least partly so.[1]

The truly significant tendency in modern metaphysics, whether it be idealistic or realistic, is toward breaking through the old fixed categories of the mental and the physical. We are returning to the point of view of the ancients, having suffered for three centuries from the blindness of the Cartesian dualism. If a chasm is opened, as it was by Descartes, between the physical and the mental, there is no way of closing it. We have struggled vainly throughout most of modern philosophy to solve a problem which, as it is stated, is insoluble: that of the relation of "thinking substance" to "extended substance." In psychology it has appeared as the mind-body problem: How can a physical thing be related to, act upon, control or be controlled by, a non-physical thing? And there is no answer in terms of the Cartesian concepts. The theory of psycho-physical parallelism merely restates the problem in a more vivid way. Psychologists have learned that they must go back to earlier notions and merge "body" and "mind" in larger concepts such as "structure" and "function." Mind and body are aspects of, abstractions from, a known reality which is wider and richer than either. In the theory of knowledge, the dualism of the mental and the physical has led to that extraordinary difficulty — how can one know the physical world? — a question which, rightly, did not enter Aristotle's head. Descartes solved it by a *tour de force*, but it would not have needed to be solved unless he had assumed that physical objects are so completely disparate from minds, so wholly resident in another universe, that nothing short of a miracle could bring them together. Physical objects are not so

[1] See E. B. Holt, *The Concept of Consciousness* (1914), chs. 8, 9. Mr. Norman Kemp Smith's recent *Prolegomena to an Idealist Theory of Knowledge* (1924), points out that the idealism, realism, and naturalism of current philosophy are indistinguishable on many fundamental issues.

foreign to minds that they cannot be known, nor are they so akin to minds that they cannot *be* when they are not known.

This is what both idealists and realists recognize, that the knowing relation is an entrance of the mind into external objects, or an entrance of external objects into the mind — whichever way one chooses to put it.

Idealism as it appears in Berkeley is a polemic against materialism rather than against realism.[1] In his day materialism was showing its head everywhere, and the good Bishop of Cloyne wished to refute a doctrine against which all that has been said above concerning idealism can be equally well maintained. To speak of reality as "matter" is to divest the term "material" (or "physical") of its specific meaning; for within this so-called "material" reality mind arises, at least as an epiphenomenon — a shadow of a shadow of matter. And this subtler thinking matter is no less a reality than the cruder unthinking matter. We have "matter $_1$" and "matter $_2$." Thus "reality," following the inevitable logic of the meaning of this term, widens out and transcends its physical as well as its mental aspect.

VIII

The capital crime of metaphysics is the attempt to reduce reality to one of its aspects or modes, a crime which is equalled only by its converse — the attempt to sever reality from all of its aspects or modes. Reality is fuller than any single set of relations, laws, principles, which can be discovered within it. It is all these, and more. But though it is not possible to define reality — to state exhaustively in conceptual terms exactly what it is to be real — there are perfectly general truths which hold without qualification for reality. Any one who believes that a rational

[1] We mean "realism" in the modern rather than the mediaeval sense. Berkeley's repudiation of abstract ideas can certainly be construed as a polemic against the realism of the schools.

metaphysics, or for that matter rational knowledge, is possible must believe that there are such general truths. The terms in which these truths are stated must be sufficiently wide to include both the mental and the physical; they must spring from the observation that reality is neither physical nor mental exclusively, but that it can be both. The physical and the mental must be capable of fusion. Further, it must be recognized that the dichotomy of the physical and the mental does not exhaust reality, but that there are other orders of being — perhaps, as Spinoza thought, an infinity of such orders.[1]

No metaphysics can successfully maintain that reality is essentially disparate from mind. Non-mental objects must be capable of coming into the knowing relation, and hence there must be unities which include both the mental and the non-mental. Nor can a metaphysics maintain that reality is essentially internal to mind in any significant sense of the term "mind." "Idealism" and "spiritualism" suggest the latter point of view, while "realism," by a modern perversion of its meaning — by stressing the externality of terms and relations, of mind and its objects, and forgetting that all relations also enter into and modify their terms — suggests the former. There should be a name for the metaphysics which consciously repudiates both of these points of view and sets out from the notion that reality is an unbroken whole whose parts mutually if not completely determine one another, the mental and the non-mental being phases within this whole. Our answer to the question — how is the mind related to real objects in the act of knowledge? — rests then on this premise, that reality is neither essentially disparate from or internal to mind. Concrete knowl-

[1] Mr. A. N. Whitehead's recent lectures at Harvard have, for the present writer, thrown a wholly fresh light on metaphysical problems; especially on this question of how, in detail, a set of general concepts can be framed which embrace the physical, the organic, the mental, and at the same time leave room for other orders of being.

edge is first of all knowledge of the real, and only secondarily knowledge of the mental or the physical. Within this apprehended reality, two orders can be distinguished, the mental and the physical, but the one is no more essentially real than the other; each is a mode of reality, bearing on itself the stamp of being.

Furthermore, the act of knowing has both its passive and active side; here as in physics the law of action and reaction applies. The mind receives an impression and correlates it with other impressions, fits it under concepts, actively assimilates it to the whole body of knowledge. Objects are perceived through thought and thought through perception, for as Kant's phrase runs, "percepts without concepts are blind, and concepts without percepts are empty." Yet the activity of thought, rather than estranging us from external reality, brings us into contact with a reality other than thought.

IX

Let us examine more fully what is meant by the "mental" with a view to showing how the mental can be continuous with the non-mental in cognition, and why thought-activity is needed to put us in touch with objects.

Forget as completely as may be the distinction between the physical and the mental; put yourself in the most naïve possible frame of mind and gaze on the things about you. What you see before you is a field of objects, probably a changing field; you close your eyes and see other, fainter objects, or perhaps only a dark area illuminated by dim streaks of light; you open your eyes, and perceive objects closely if not indistinguishably like those you first saw; and all the while within your body you experience faint or vivid sensations of movement, perhaps of pain or of a vague well-being. There is no reason for singling out any part of this whole experience as mental. Some parts of it are

more closely associated with the body than others; the pain is localized in the body, while the wall of the room is not; but externality or internality to the organism gives us no criterion of the mental.

The distinction is certainly not an early datum of experience; it comes to the surface only after reflection, only after certain sequences and orders of events have been observed. Then it is discovered that some of the things experienced hang together by a different sort of coherence from others. Within the single field of real objects, which is the original cognitum, we find two worlds which differ in structure but easily become a single world, and there can be no question as to how knowledge passes from the one to the other. From the beginning knowledge is synoptic, it covers both.

The lineage of this view, which was strikingly put by William James in his essay, *Does Consciousness Exist*,[1] can be traced at least to Spinoza: "thinking substance and extended substance are one and the same thing, which is now comprehended through this and now through that attribute." [2] James need only have added to the two types of structure found in the world of "pure experience," an infinity of types of structure (corresponding to the infinite attributes) to have become a thorough-going Spinozist. Mr. Bertrand Russell, who has elaborated James's view, gives the name "mnemic causation," as distinguished from "physical causation," to the type of law which is characteristic of the mental realm.[3]

Memory is certainly one of the chief principles of order in the mental world, but the term "mnemic causation" is too suggestive of Hume. Its effect is to turn the attention away from the very essense of mind, activity toward an end — the unity of

[1] W. James, *Essays in Radical Empiricism*, 1912.
[2] B. de Spinoza, *Ethics*, Bk. II, Prop. VII, note.
[3] B. Russell, *The Analysis of Mind* (1921), sect. 4.

memory and purpose, of backward and forward looking intentions, which is a personality or self. If the soul-substance view of mind is dead, so also is that of Hume — that the mind is a stage where ideas pass and repass without a spectator to take account of them. The stuff of "pure experience" does not combine and recombine of its own weight, according to the principles of association or of mnemic causation, to form a mind. The laws of mind are laws of purposive activity; memory is itself an activity; will, striving, conation, are at the core of mind; and these strands of activity are always caught together in a unity which is the self of the moment. This self changes, but not without persistence of its structure. At any moment a definite past, which can be called up and known again (though not exactly as before), converges toward the present, and a future — purposed, desired, intended, striven for — expands from the present. This is a mind.

Moreover, the activity of the mind can become reflexive, it can be turned upon itself or its own products; and this "reflection" occurs in two very different ways. (1) In desiring, we know that we are desiring; in knowing, we know that we are knowing. Josiah Royce speaks of this as the "self-representative" power of knowledge,[1] that is, knowledge can reflect on its own processes. (2) The activity of the mind tends also to crystallize into a content which is still wholly within the mind. To think of or intend something is to set in motion the whole apparatus of cognition; the thought refuses to continue merely as a pure intention but strives to become concrete, that is, to fulfill itself; and thus an image or "reflexive content" appears. The image is a deposit of the activity. To know the image is therefore not merely to know that we are knowing; it is to be aware of something the mind creates. Images are the products of minds rather than minds themselves; they arise in the process of

[1] J. Royce, *The World and the Individual* (1901), Vol. II, pp. 509 ff.

mental activity, but they are not the activity, nor are they of the same stuff as non-mental objects. Though the activity of the mind creates reflexive contents, there is no foundation for the belief that all experience is of reflexive contents only; and if this is the meaning of Locke's statement that "knowledge is conversant only with ideas," an essential fact of knowledge is overlooked, that one knows ideas and the mind that creates them only by distinguishing them in a reality which extends beyond both.

Given, then, that the mind from the outset knows something other than itself, and it still remains to be shown how the mind is continuous with this non-mental reality in cognition.

This continuity is not identity. The mind is other than the external reality it knows; there is always more of the reality to be known, and always the *knowing*, which is of the mind but not the same as the non-mental object. (The *knowing* tends also to create a reflexive content which fuses with the non-mental object, but, difficult as these images are to distinguish from the object, they are not identical with this object.) The epistemological monism which identifies mind with its non-mental objects makes the object of knowledge a sponge that absorbs everything — error and illusion, as well as meaning and truth.

Mind is cognitively continuous with non-mental objects in the same general sense as other things in this world are continuous with one another. And we are not speaking here of the mathematical theory of continuity, but of something found in experience, which M. Bergson describes as "interpenetration" and Mr. Whitehead as the general "togetherness" of objects and events. This continuity is not, we believe, a relation. It is rather the unity or wholeness within which both terms and relations arise. Distinct things singled out as self-identical always merge in the wholes within which they are distinguished, and this union

of the parts in the whole is not due to confusion in the perception that distinguishes them, but to the nature of reality. Though reality at all points takes on distinct structures or forms, these structures are always elements in wider structures, and every whole is continuous with other wholes.

Let us recall what was said previously in connection with *relations*.[1] A is related to B not because the relation R which unites A and B is related to its terms, for if this were so an infinity of relations would be needed to bring about a unity of the terms. The terms and relation are joined once for all. They form a whole in which the aspects A, B, and R can be distinguished; this is what we mean by saying the relation R holds between A and B. There is no point where the relation ceases to be a relation and becomes a term, or where the terms cease to be terms and become a relation. Motion is an apparent case of such continuity. There is no point at which the moving object ceases to be in one place and passes into another; in fact, the passage is just a continuous process which cannot be completely described in terms of places or points, any more than a relation between terms can be described in terms of elements and a relation. We must have the ultimate concept of the unity of the elements. We must think of the elements as abstracted from this unity, instead of thinking of the unity as added to the elements. We must think of the points and instants, in terms of which we describe the motion, as abstracted from the passage, rather than of the passage as superimposed on the points and instants.

The mind and its objects fuse as one segment of a motion fuses with another, as a relation merges with its terms, as any part of a whole or any whole, with another. The mind projects itself into the non-mental and the non-mental into the mind. There is an unbroken flow of process, and throughout a stretch of this process—at the segment of "greatest luminosity"—the

[1] See above, ch. III, sec. vii.

cognition of the object comes into being. Minds are together with external objects in the same general way as objects, events, situations, are together with one another.

This continuity of mind and its objects needs, however, to be further specified. Continuity (or unity) is a general background against which *any* elements of reality — whether they be minds, physical objects, or what not — stand out; and in each distinguishable whole of experience this continuity is of a specific sort. Thus the whole (*ARB*) might be a whole of "spatial beforeness", that is, it might be the fact "*A* is before *B* in space." Motion is obviously a spatio-temporal unity of a specific kind. What sort of unity or continuity is this cognition of a non-mental object? Certainly it is not a spatial continuity. Though the object known may be spatial, the cognition is not *in* space but *of* space. These two types of whole — the spatial and the non-spatial — come together in the wider whole of cognition. On the other hand, the continuity of mind and its non-mental objects is both *in* and *of* time; the mind as well as the world it cognizes is a changing, temporal unity. In the specious present of knowledge we grasp, in an act which is itself temporal, the immediate past and future of the thing known, so that each whole of cognition — being itself in time — is nevertheless a survey of time. And yet the object of knowledge has its non-temporal aspect. It is a "what," a universal as well as an individual, and its "what" cuts across time as well as across space and belongs intrinsically to no single time or space. This is indeed the greatest paradox of knowledge, that being in time it takes hold of both the temporal and the non-temporal. The key to this paradox is memory. In so far as we attain to any knowledge of universals, of timeless objects, we do so through memory; and since concrete knowing is always the seizure of something both universal and individual, of something unique and passing which is nevertheless blended with something that has been and may

be again — for this reason, concrete knowing is a continuity of a remembered past with a new, yet familiar present.

The passage of thought is plainly not motion, though it is activity; and one speaks only metaphorically of the mind as "going forth to meet its object." Knowing is a fulfillment, a realization, in which the mind attains an end which has been obscurely before it; and whether or not teleological categories are of use in describing nature, they are indispensable in describing mind.

Finally, the act of knowledge is inclusive of itself. In knowing objects we know the process by which we know, the mind reveals itself through its commerce with its non-mental environment. And beyond all this, *knowing* has its peculiar tang which is no more subject to description than the sound of a tonic triad or a chord of the seventh. Analysis discerns in *knowing*, as in the chord of the seventh, a number of phases, but it still remains simply what it is — a unique union of mind and object.

X

It must not be supposed that the external objects which project themselves into knowledge are those that act on the senses. We perceive colors, shapes, smells, sounds, motions, houses, trees; we observe relations between these objects and laws that hold for them; but we do not perceive light-waves or sound-waves, molecules, electrons or electronic structures, nor do we perceive the bodily processes which light-waves or sound-waves, molecules or electrons, arouse in us. One can construct a theory of how these infra-experiential entities act on the sense-organs, but it is not possible to perceive these entities through their action on these organs.

An immense amount of error has been propagated by the view that the only real physical objects — or, for that matter, the only real objects — are these infra-experiential entities in

terms of which physical theory is constructed. The problem of our knowledge of physical reality is totally misconceived if one attempts to solve it by explaining how these entities, which are undoubtedly connected in some way with our sensory reactions and with all our intellectual processes, can be known through this connection. Whatever the relation of these entities to mental processes may be, it is certainly not the cognitive relation. This must be sought elsewhere.

The physical objects of concrete knowledge must be distinguished from the physical objects of scientific theory; these two types of physical object together make up the complete physical object. There is no doubt that perception penetrates only a short way into the complete physical object; what is perceived must be pieced out by a theory as to the nature of this whole object. But this lack of concrete knowledge of the whole physical object does not make the objects of perception purely mental. They retain their otherness, they still belong to an order different from that of mind. The presented physical object is a mediating link between the mind and the physical object of scientific theory. It is sufficiently like the mind's reflexive content (images, dream-perceptions, etc.), to be deceptive when taken out of its setting. (There is no intrinsic difference for perception between an isolated physical sound, for example, and an isolated sound-image.) Yet the presented physical object, in its setting, bears the marks of a structure different from that of the mind or the mind's reflexive contents; and it is on this observed difference of structure that a theoretical world of physical objects, extending completely beyond concrete knowledge, is built. In the act of cognition, we are aware at once of the distinctness and the unity of our minds with the *presented* physical object; but we are never concretely aware of how this perceptual object, or the mind, is related to the *scientific* physical objects. A color is doubtless vastly unlike a light-wave, yet the color is a physical

phenomenon, a non-mental reality. It comes into being in a certain situation, theoretically represented as a light-wave striking an eye. It is a part of the whole physical situation of which the light-wave is a part; it is that part of the situation which can *be* in knowledge and at the same time in the external world.

The problem of how the objects of physical theory are related through the presented physical objects to the mind, of how these three can be together, is a phase of the problem of mind and body. But the problem of the cognition of the physical world, or for that matter of any order of being beyond our own minds, is wholly another question. If there is to be any physical theory and any problem of mind and body, both the physical and the mental must be known. The mind could not leap miraculously to the notion of an infra-experiential world of atoms and electrons and then ask, "What is the relation of this world to the mind?" Whatever the solution to the mind-body problem may be, the mental and the physical are continuous in a specific way in cognition. They must also be continuous in some other specific way through the action of molecules, electrons, light-waves, on the nervous system.

Since the presented physical object arises in a situation in which both a mind and other physical objects, not presented, coöperate, one can ask — what aspects of the presented object are relative to this situation? — and what aspects extend beyond it?

These questions can be generalized for any situation. Every situation is altered in general complexion if some of its elements are removed. Something more than these elements goes out of existence; the original "what" of the whole is extinguished and the remaining whole is a new "what," perhaps of a very different complexion. Thus there is something in every situation which is relative simply to that situation. Take away the spires of the cathedral at Chartres and the proportions of the building

are destroyed; new, probably ugly, proportions come into being. It is not simply that the spires are missing; the personality of the cathedral has vanished, to be replaced by another. And yet the two personalities have something in common; the mutilated whole can be pieced out to give the original; there are hints as to what the old personality might have been. So also in every whole there are suggestions of other wholes of which it might be a part.

The presented physical object, issuing as it does from the unity of a mind and other physical objects not presented, must be a distinct "what" relative to this situation. But to call attention to the fact that its general complexion is determined by the psycho-physical situation is not to deprive the presented physical object of its reality; nor does it make the object any more essentially relative to mind than to the physical world beyond. Just as a real object other than the single notes comes into existence when a chord in C-major is sounded, so a real (perceptual) physical object comes into existence in the proper situation.[1]

What is the nature of the physical object when it is not perceived? All one can answer is that it cannot be exactly the same as when it is perceived. The situation has altered. One element, the element that makes the object *perceptual*, is absent; but just what aspects of the object pass out of existence with the absence of perception we certainly cannot say. Locke describes the un-

[1] The question of the relevance of the perceptual object to a psycho-physical situation is not the same as that of the "privacy" of this object for an individual perceiver. Every real situation is individual, and this individuality is determined by that of its elements; hence, there must be an individuality in the perceptual object which is bound up with (though not reducible to) the individuality of the perceiving mind. But every situation has, also, its universal aspects, so that the "privacy" of the perceptual object is not incompatible with its "publicity." Indeed, as has been shown in the discussion of the individual (above, ch. III), only the public aspects of the object are clearly and determinately known. The perceptions that men *share* are those of general natures and relations, *i.e.*, of red, motion, etc., rather than of natures and relations as individualized in particular situations, The question of the "privacy" of the perceptual object is one phase of the more general metaphysical question — how can a real object be at once individual and universal, absolutely unique yet like other real objects? See below, secs. xii and xiii.

perceived physical object as colorless, tasteless, odorless, soundless, but insists that it retains its geometrical and mechanical properties. Yet even Locke admits that the colors, smells, tastes, odors, are still *in* the physical object as "powers", that is, there is something in the unperceived physical situation which becomes a color, taste, smell, or odor when perception is added. Perhaps an unperceived color differs much more widely from a perceived color than an unperceived motion differs from a perceived motion, but this supposition would not warrant us (following the distinction between primary and secondary qualities) in placing some of the modes of being of the perceptual object only in the mind and others both in the mind and in the physical field beyond the mind. Certainly the whole object as unperceived is continuous with the object as perceived; the latter is a real whole merging with another (unperceived) real whole and giving indications as to its nature.

Thus, if there is a "somewhat" in the perceptual object which is peculiar to it, there are also a vast number of properties and relations which are not peculiar to it, but which stretch out into the unperceived reality. For all reality is shot through and through with the general. Though each real whole is unique, it is still an instance of many universals which link it to other wholes. Physical theory seizes on certain relations and properties found in the perceptual physical object,— for example, motion, acceleration, mass, spatial relations, etc., — and by refining and extending these properties and relations, so that they can be used as instruments of exact description, pieces out the picture of the whole physical reality.[1]

[1] Along with the tendency of the mind to coöperate with the elements of the physical situation in giving birth to the perceptual physical object goes an opposite tendency — the tendency not to coöperate. Certain elements of the physical situation are excluded from the perceptual physical object. Thus, not only is there the problem of how the unperceived color or motion is perceived, but there is also the problem of why the light-waves, electrons, molecules, etc., are *not* perceived.

XI

What has been said of the relation of the mind to its non-mental objects makes it even more difficult to believe that these objects are known passively by impressing themselves on the mind. Both the mind and a perceptual (external) object are together in the earliest wholes of cognition. The object is in the mind, the mind is in the object, and there is no reason to think that the impulse to a more complete cognition comes — as the blank-sheet view of mind would have it — from the object rather than from the mind. It is most unlikely that the mere repetition of a situation in which a mind and a perceptual object are together leads to a clearer knowledge of the object. Some other factor must also be at work. The object must catch the attention; it must attract as well as impress the mind, and this means that it must complete the mind's intention.

Cognition is always the fulfillment of an intention, and so there must be native impulses to cognition from which the first knowledge of objects springs. The mind no less than the body must have its initial structure, which grows and is modified by successive acts of knowledge. And the structure of the mind, like that of the body, must be adapted to its environment. This is — if one wishes so to speak of it — a theory of innate ideas; but just as most criticisms of the "instinct psychology" object to the specificity of the instincts assumed rather than to the fact that there are instincts, so one can repudiate innate ideas so special as those of God, of right and wrong, of mathematical truth, without repudiating all innate cognitive tendencies.

An original impulse toward analysis, toward selection or sifting, is found in knowledge. To cognize anything clearly is to isolate it from its surroundings. This tendency toward analysis is accompanied by another, the tendency to hold together with its surroundings what has been sifted out. Not only is a single

element more clearly known through selection, but the whole of which it is a part is also more clearly known. And these most general impulses are associated with more special ones, such as jumping at a noise, turning the eyes toward a light, grasping, etc. Indeed, every special instinct is cognitive in that it leads us to analyze and to take account of the unity of our environment. Objects do not of their own accord separate themselves out from experience; they might *impress* themselves forever on the mind without giving rise to the dissociation of their parts which is analysis. Yet reality is so put together that it permits this dismemberment and also resists it, thus satisfying both the impulse to analysis and to unification. The mind finds real chinks and crannies in the object, real lines of stratification along which it breaks up for thought, so that we do not create out of whole cloth a mere appearance of analysis. We adjust ourselves through analysis to something real in the world beyond. (The reason why knowledge is for Kant an act of introversion is that the cognitive impulses are totally out of adjustment to reality; there is no trace of agreement between the way in which the mind tends to think and the way in which objects *are* beyond the mind. It would be strange if, with bodies so nicely adapted to the physical world, our mental impulses were completely askew to reality. The theory of evolution ought to apply to minds no less than to organisms.)

Analysis is a cumulative process; what has been analyzed can be further analyzed, one analysis makes possible another. We select from what has been selected, we learn that things are together which have not been previously observed to be together, the channels of cognition become more and more complex, more and more numerous, enabling us in the end to take account of subtleties, distinctions, unities, which were before obscured.

Finally, we become aware of the widest intentions of thought, which make analysis possible. These are the intention to direct

every intention toward one and only one object (the principle of identity as a rule of symbolism); the intention to direct distinct intentions toward distinct objects (the principle of contradiction as a rule of symbolism);[1] and the intention to apprehend facts, situations, objects, as unified groups of distinct elements. We discover that reality yields to these intentions, that every object is what it is, self-identical and distinct from other objects, yet joined with them in continuous wholes. The mind thus learns to be conscious of its categories, its structure, through the operation of these categories, the functioning of this structure, in concrete knowing. At the same time its contacts with special aspects of its environment multiply and refine its more special concepts.

But the activity of the mind in the grooves of concepts would not bring us into touch with reality unless reality were permeated by the general. This fact is what adapts the environment to our cognitive impulses. Every concrete object is at once universal and individual, absolutely unique and yet like other objects. It is the *universal* — which is itself an object — which fulfills our intentions in knowledge. For, as has been pointed out in another place,[2] analysis is first of all the dissection of an object into a "what" and a "this," a universal and an individual aspect. If there were nothing in the real which persisted, if every object came once and never again, our concepts could take hold of nothing. Reality would slip by unanalyzed, taken account of only in that pure awareness which lies below the level of clear cognition. No situation is wholly new; hence, it can be assimilated to knowledge held in the memory, the mind can come to rest in the apprehension of something which completes its intentions. In any moment of time (and knowledge is both *in* and *of* time) the mind, through the fulfillment of intentions

[1] See above, ch. VI, sec. iv.
[2] See above, ch. III, sec. i.

which bridge many times, attains a knowledge of a reality stretching far beyond the moment. With the conceptual tools forged by the aid of cruder tools, the mind digs into the object before it to discover principles which radiate into other objects — and into all reality. Thus the immediate impression is the *occasion* of true knowledge; yet knowledge in expanding beyond the moment does not turn upon itself to a world of appearances. It grows always in the direction of the real.

In all this something which remains to be considered has been anticipated. It has been assumed that reality has a structure. The essence of structure is logical form. Does reality exhibit logical form? Is reality made up of self-identical and distinct objects which coalesce into wholes (groups) with elements of unity and terms? Are universals, that is, elements of unity, real? Do the principles of identity and contradiction, which are the formal conditions of the *being* of self-identical and distinct objects, apply to reality?

If these questions must be answered in the negative, not only does the activity of thought end in a blind-alley, but, from the metaphysical point of view, the truth which we have described as the correspondence of concepts through their structure with real objects becomes error. If reality is structureless, it cannot be adequately known through a thought bound to logical forms.

XII

Here is a turning-point in any metaphysics of knowledge. How are we to stand off from thought and hold it up beside its objects to discover whether the two are commensurate? If they are commensurate, if there can be a real identity of structure in concepts and the things they refer to, we cannot *demonstrate* through concepts that such an identity is possible, for this correspondence of concepts with objects is itself the condition of rational truth and demonstration. Nor can we know this struc-

tured reality in a cognitive act which takes no account of structure. And if thought and its objects are incommensurate, we can not prove them to be so by going back to the negative arguments of agnosticism. We cannot, through the logical forms of thought, show that reality is without logical form; some positive knowledge, not through thought, of this formless reality would be needed.

Beside agnosticism, which on purely negative grounds denies reason access to reality, stands positive irrationalism — a metaphysics which condemns reason on the strength of an intuition that goes deeper than thought to a mystic union with the real. Agnosticism does not successfully negate rational metaphysics. Does this positive irrationalism meet with better success? To answer this question, we must consider in what sense the act of cognition as a whole escapes from the concepts which at once confine and guide it.

Every act of concrete knowing overflows the conceptual. To perceive an object is to capture in experience what one intends, but it is also to capture more than one intends. This is one reason why the real is alien to thought. Concepts can be multiplied indefinitely, yet the real object to which they lead is not exhausted and is always ready to yield further analyses, to submit to new descriptions. I conceive the object as a unique "somewhat" in certain relations to other unique "somewhats"; but when I grasp the object concretely I know that these concepts direct me only to some of its aspects. The object *is* as I conceive it, but is also other than I conceive it; and in knowing that what can be picked out clearly and stated as true of the object is only an aspect, I must apprehend the object as transcending and including all its aspects.

What has been said before of the universal and the individual sharpens this point.[1] For conceptual knowledge, the individual

[1] See above, ch. III, secs. ii, iii, and iv.

is always attempting to become universal by dissolving into a complex infinity of characters and relations, but it succeeds only in being an indeterminate x; while the universal is always seeking to become individual by qualifying and requalifying itself in an infinity of ways. If I wish to conceptualize the individuality of a pebble picked up on the beach, I am driven to hundreds of characterizations, of measurements, of minute observations; yet the pebble is still the x of which these relations and qualities can be predicated, and I cannot even be certain (with Leibniz) that an infinity of such predicates would distinguish *this* pebble. I cannot, in thought alone, bridge the gap between the x, which means indeterminately an individual, and the other concepts, which stand for its predicates and relations. It is in vain that I fill in with more predicates. In concrete knowledge the gap is closed, though not by concepts. I know the pebble, hold it in my hand, as a unique object in which all the predicates I have observed are reconciled. It is not merely that concrete knowledge adds sensation to thought; I do not *sense* the unity of the predicates in the individual. The act of knowing has pushed out beyond sensation and thought to an *intuition* of the object. This intuition has been spoken of before as *pure awareness*. Pure awareness issues from thought and sensation; we cannot catch it in itself apart from the other phases of cognition, and yet it is not the same as either of these other phases. What intuition gives us is a residue of knowledge, left over when all that is clearly conceived or sensed in the object has been analyzed away.

Consider the knowledge of a personality. Here intuition is especially prominent. Calculation, analysis, will carry one only a short way in adjusting himself to persons; something like talent, skill, tact, is needed — not a talent separated from the power of analysis, but a talent which makes use of every shred of clear knowledge, and yet does not rely wholly on clear knowl-

edge. To know a personality is to enter into that personality, not blindly, but so that analysis plays its part and is left behind.

Pure awareness is not sensation for the reason that a sensation is atomic; it is a distinct "what," *e.g.*, this redness, this sourness, distinguished within a perceived whole. Moreover, a "pure sensation" would be a sensation of nothing whatsoever, for sensation cannot be thinned down till every vestige of meaning, thought, has been wiped away. When this search for a "pure sensation" is pursued, the sensum approaches a vanishing point. A "pure sensation" is the thinnest of abstractions, an imaginary limit which cognition never reaches. When knowledge overflows the conceptual, it does not do so in the direction of sensation. Nor does knowledge pass beyond concepts by going to the other extreme, by sifting out all traces of sensation, leaving the materials of experience behind and attaining an esoteric vision of a reality that never was and never could be found in perception. Yet to speak of pure awareness, of non-conceptual knowledge, is to suggest the one or the other of these views — sensationalism, or rationalism become esoteric.

An act of concrete knowing is bathed in an atmosphere of the non-conceptual; it has a part of its being in a medium which is neither pure thought nor pure sensation; and no act of concrete knowing is complete until it finds itself surrounded and upheld by the non-conceptual. This ever-present background of pure awareness tends, like other backgrounds, to be forgotten and to emerge only when a foreground has been clearly distinguished. But it is always there, bringing thought and sensation together and completing knowledge as neither thought nor sensation can. It is impossible to isolate thought from sensation, and equally impossible to isolate pure awareness from sensation *and* thought. Perceptual thought transforms itself into pure awareness and pure awareness passes smoothly into perceptual thought. There

is no pure awareness dissociated from these other modes of cognition.

An anti-intellectualistic philosophy such as M. Bergson's wishes to set pure awareness or intuition on its own feet, to sever rational thought from the non-conceptual medium into which and from which it flows. M. Bergson does violence to the cognitive act; he looks at cognition abstractly in the very effort to fasten on that which is most concrete in it. Instead of seeing it as it is, a fusion of reason, sensation, and intuition, he exalts the irrational component at the expense of the rational.

"By intuition," M. Bergson explains, "is meant the kind of *intellectual sympathy* by which one places oneself within an object in order to coincide with what is unique in it and consequently inexpressible. Analysis, on the contrary, is the operation which reduces the object to elements already known, that is, to elements common both to itself and to other objects. To analyze, therefore, is to express a thing as a function of something other than itself. All analysis is thus a translation, a development into symbols, a representation taken from successive points of view from which we note as many resemblances as possible between the new object which we are studying and others which we believe we know already. In its eternally unsatisfied desire to embrace the object around which it is compelled to turn, analysis multiplies without end the number of its points of view in order to complete its always incomplete represnetation, and ceaselessly varies its symbols that it may perfect the always imperfect translation. It goes on, therefore, to infinity. But intuition is a simple act." [1]

M. Bergson's writings, like his snowball rolling down hill and growing as it rolls, bulk together to force it on the mind that concrete knowing is more than analysis. But that analysis is es-

[1] H. Bergson, *An Introduction to Metaphysics*, T. E. Hulme's translation (1912), p. 8.

sentially falsification, that intuition enters into reality to find a changing, formless, flowing *durée* which eludes all concepts — these are conclusions which follow only from making a part of concrete knowing equal to the whole. M. Bergson is Heracleitus with the *logos* left out. If reality is change, this fact, as Heracleitus observed, does not itself change, and here is an entering wedge for the permanent. An intuition cut off from concepts could not name the reality it knows. Just as the concrete content of pure sensation or of pure thought would approach a zero, so the concrete content of pure intuition tends to be nothing. M. Bergson insists, for example, that to conceive a motion as an infinite series of brief motions taking place at point-instants arranged in order is to fail to know the passage which is the essence of the motion. One must identify himself with this passage by intuition. Yet this pure intuition of the wholeness of the motion is, in itself, no less empty than the representation in terms of point-instants. Though there is something real in the passage which is given only in intuition, there is also something real in it which corresponds to the conceptual series of point-instants.[1] To know the motion either simply as *passage* or simply as a related series of motions at point-instants is, in each case, to have an insecure hold on its reality.

Irrationalism springs from a desire deeply rooted in human nature — the desire to give oneself up, like a resting swimmer on a bright sea, to the sustaining buoyancy of a limitless and intimate reality. But one knows the buoyancy of the sea only by learning to swim, and one knows reality only by learning to think out his intuitions and to gain new intuitions through his thought.

M. Bergson's feeling that the deliverances of intuition prove

[1] See A. N. Whitehead, *The Principles of Natural Knowledge* (1919), chs. 8, 9, 10, 11 on extensive abstraction for a definition of points and instants which correlates these entities with the perceived reality.

the inadequacy of thought cannot rest on a contradiction between what is known without concepts and what is known in concepts, for *contradiction* has meaning only where concepts apply. Intuition could not contradict reason. When the cognitive act is taken in its wholeness as the convergence of thought, sensation, and intuition upon an object, this feeling of the inadequacy of thought to reality melts away. Though one is aware that reality is not completely compressed into the molds of reason, he still finds something in the real which fits these molds. Reality exhibits logical form.

XIII

That reality is logical in form means that the distinction between terms and the universals, that is, the qualities and relations, which modify and unite these terms, is a real distinction. It means that there are basic terms, individuals, about which qualities and relations cluster to form concrete facts. It means that every object, whether individual or universal, is self-identical and distinct from other objects, and finally that these objects unite into wholes, groups, which are themselves real objects.

For Kant it is just this logical form which is created by thought and which confines thought to a world of phenomena; and for M. Bergson this logical form is the product of a platonizing intellect. But since the mind from the outset of knowledge is in contact with the real, in knowing objects it must be able to know that logical form is *in* reality and not merely in a mind compelled to think in this fashion. This is the first and most fundamental metaphysical insight.

Let us return to the *vis sua nativa* of the intellect of which Spinoza speaks. If there is any knowledge, there is a metaphysical insight which upholds knowledge at every stage and which becomes conscious of its own general intentions and, through

these, of the general structure of reality, when knowledge is faced with the question of its own validity. To know is, from the beginning, to have an insight into the real. This power is not attained after long meditation. It is not by falling into antinomies and seeking a way out that thought is enabled finally to seize reality; nor is it by dismembering the act of knowledge and attempting to know through pure sensation, pure reason, or pure intuition. This power is a crude and original endowment of the mind. It is manifest in the simplest act of concrete knowing, not as one component but as the very life of the act itself. To reflect on knowledge, to ask how we can know and what we can know, is to bring this original metaphysical insight to a consciousness of itself. Instead of being aware of objects as fulfilling specific intentions, as being of this or that sort, we become aware of objects as fulfilling the most general intentions of thought, and thus we discover the logical structure of the reality to which our thought is from the first adapted, and from which it is never cut off. We cannot step outside the act of knowledge to an intuition which renders all knowledge but itself invalid.

Knowledge from its inception embraces a reality which is logical in form, without being aware of what constitutes logical form. We do not pause to reflect that in knowing we are distinguishing terms from their qualities and relations, that we are taking some terms as individual, that we are treating objects as self-identical and distinct, though as united into real wholes. But if we do pause to reflect, we know that only because objects conform to these highly general conditions are they real. To know that the widest intentions of thought are fulfilled by objects is to tighten rather than loosen the native grasp of the mind on reality. In perceiving the logical framework of the reality within which it operates, the mind knows its own power with a new clarity.

That concrete knowing even of the most elementary sort is

insight into the real, and that the act of knowing as a whole is the source of this insight, are the basic propositions of a metaphysics which justifies knowledge. These propositions do not assert that all concrete knowledge is true. An object may not *be* as it is known, it may be misconceived. But even a misconceived object *is*, if it is known concretely; it is there to be reckoned with, to be fitted into the scheme of reality. The act of knowledge, becoming conscious of itself, reveals the essential pattern of this scheme, but it does not reach through more readily to the details than it would without this self-consciousness. By discovering that the general principles of knowledge are rooted in the general structure of the real, we learn that a rational truth is possible, but we do not establish specific truths. Specific truths must be hewn out laboriously from the materials of experience by many cumulating acts of knowing. The details must be fitted into the general pattern, and for this the skill which grows with the expansion of knowledge is needed.

Thus in one direction knowledge is complete, it can take secure hold of the form of the real; while in another direction it is never complete, it is being continually enlarged and fortified by new details.

Within the essential scheme of the real, there is room for an infinite variety of detail. The form is *in* the detail; the variety of detail, *in* the form. The metaphysical insight which makes use of the complete cognitive act shows us how passing and temporary objects embody universal principles and yet are themselves unique products of an ever-renewed process of becoming. Reality is as changeful and growing as M. Bergson finds it to be, but the change is an affirmation rather than a denial of logical form. Against the newness, the individuality, of the thing of the moment, the universal, which is neither new nor individual, stands out. M. Bergson condemns analysis because it must "express a thing as a function of something other than itself" and

so miss what is unique in the thing. One can condemn pure intuition because, in refusing to express a thing as a function of something other than itself, it misses what is not unique in the thing — and so fails to know this very uniqueness as sharply as might be if analysis were joined to intuition.

To the irrationalist, reality cannot be self-contradictory for this reason, that it resists concepts — and contradiction is a logical relation between concepts; the term has no meaning for him as applied to the real. The reality known through the fuller metaphysical insight of which we are speaking is non-contradictory in another sense; the principles of identity and contradiction can be truly affirmed of it. This reality can be consistently presented and represented through concepts. And yet, if we consider these principles themselves, it is plain that they permit an endless mutability of the particular. As statements about the real world they affirm that "any object is identical with itself and distinct from other objects," but they give us no information as to what objects are self-identical and distinct. They tell us merely that, however tangled the lines of identity and distinctness may be, there must still be such lines. Consistency has thus its metaphysical basis as a test of truth which is valid even though no experience includes a reality which is complete, full-grown, in every respect. Judgments can go on correcting themselves by their consistency even though consistency does not determine the subject-matter of any truth.

This infinite perfectibility of knowledge does not lead to the corollary that there is no truth except through an infinite process of correction. We know the logical form of reality without such a process; logical form is a perfectly general aspect of the real which can be grasped in finite knowledge. Yet it is only an aspect. And if we can know one perfectly general aspect of reality in finite knowledge, we can know others — provided there are such aspects to be known.

The laws of nature seem to hang midway between these purely formal principles and particular truths. It is possible, as C. S. Peirce, H. Poincare,[1] and others have suggested, that whole systems of natural law are superseded by other systems, as one event in history is superseded by another, or old sets of personal habits by new ones. But if the universe, from time to time, gives up one general mode of behavior to adopt another, it does not contradict itself. The particular events of history can be truly known, so long as we do not insist that they are more than particular events; and so a system of natural laws, which might dissolve and pass, could be truly known if we did not believe it to be eternal — if we could see what went before and came after. But just as every set of personal habits embodies the laws of human psychology, so every one of a successive infinity of cosmic habits would embody the psychology of the universe. And even if the laws of this cosmic psychology were more concrete than the formal principles of structure we have discussed, they could be known in perfect generality.

Whatever the whole of reality may be — if there be a whole— we can be certain that we do not know this whole. A complete understanding of any whole rests on a knowledge of its parts, and a complete understanding of the parts on a knowledge of the whole, but it does not follow that there are no truths which are valid for the parts alone. Though the process of the growth, the becoming, of reality may go on infinitely, and though it may be impossible to know all there is (or will be) to be known concerning any fragment of the real — unless it be in a mind which spans this infinite process,— still these considerations do not demonstrate the impossibility of attaining truth in a knowledge which is immersed in the process of becoming.

[1] C. S. Peirce, *Chance, Love, and Logic* (1923), "The Doctrine of Necessity Examined"; also, H. Poincare, *La Valeur de la Science* (1905), ch. 11, and *Dernières Pensées* (1913), ch. 1.

XIV

No criticism of knowledge, then, can destroy knowledge; no theory of appearances can prove that reality is unknown or unknowable. For a knowledge of even the most contradictory of appearances is the beginning of an insight into the real. The theory of knowledge clarifies this insight as one clarifies his speech by learning the grammar of the language to which he is born. But without speech we could not learn the rules of speech, and without true knowledge we could not construct a theory of truth.

At the poles of knowledge, beyond clear statement and rational proof or disproof, are scepticism and mysticism. The sceptic believes nothing he can state; the mystic can state nothing he believes. Rational knowledge shares with scepticism a readiness to revise itself but refuses to join scepticism in revising itself into a state of total incredulity. Something stands firm, if it be no more than the good sense which Descartes found "is of all things among men the most equally distributed." With the irrationalist and the mystic, rational knowledge shares the conviction that reality cannot be completely packed into neat concepts, but rational knowledge cannot believe that there is no neatness in reality, that because something escapes nice formulation, everything does.

Making use of all the instruments at its command — sensation, thought, intuition — knowledge can reflect on its own processes and reaffirm, as against scepticism, mysticism, and agnosticism, its own ability to take at least partial possession of the real in a rational experience. But in order to know, it is not necessary to inquire how it is possible to know. As the lungs are prepared by nature for breathing, the hands for grasping, the legs for walking, so the mind is prepared for knowing. We may with infinite patience lay bare the anatomy of the mind, but we

cannot in doing so discover that the mind's function is to trick us or to shut us off from the real. The air the mind breathes, the substance it grasps, the ground it walks on, is reality. The theory of knowledge, like the knife of the surgeon, may be able to separate the delicately woven tissues of thought, but it cannot give or take away the power to know.

SUPPLEMENTARY READINGS

THIS list of supplementary readings is not a bibliography; it is intended to bring together for the use of the student some of the contemporary literature — chiefly in English — on the theory of knowledge, and to suggest directions in which the topics treated in the text can be expanded. It will be well for the reader to take up the selections under each heading in the order given here; the readings most closely allied to the text are placed first in each group. References to the classical philosophers are omitted, except in a few instances.

INTRODUCTION

On the relation of the theory of knowledge to metaphysics:

C. D. Broad, *Scientific Thought* (1923), Introduction.

W. T. Marvin, *The New Realism* (1912), "The Emancipation of Metaphysics from Epistemology."

S. Alexander, *Space, Time, and Deity* (1920), Introduction.

CHAPTER I, MEANING

On the psychology of meaning:

E. B. Titchener, *The Experimental Psychology of the Thought Process* (1909), Lect. 2, "Reference to Object as the Criterion of Mind," Lect. 5, "The Experimental Psychology of the Thought Process."

E. Rignano, *The Psychology of Reasoning* (1923), ch. 4, "What is Reasoning?"

K. Koffka, *The Growth of Mind* (1924), ch. 5, sec. 10, "The First Use of Language."

J. B. Watson, *Psychology from the Standpoint of a Behaviorist* (1919) ch. 9, secs. A, B, "Explicit and Implicit Language Habits."

G. F. Stout, *Analytic Psychology* (1896), bk. ii, ch. 10, "Thought and Language."

On the philosophy of meaning:

F. C. S. Schiller, B. Russell, H. H. Joachim, *Mind*, N. S. vol. 29 (1920), pp. 385 ff., "The Meaning of Meaning, a Symposium."

R. F. A. Hoernlé, *Mind*, N. S. vol. 16 (1907), pp. 70 ff., "Image, Idea, and Meaning."

C. K. Ogden and I. A. Richards, *The Meaning of Meaning* (1923), ch. 2, part 2, "Towards a Science of Symbolism," ch. 9, "The Meaning of Meaning," ch. 10, "Symbol Situations."

On mediate and immediate knowledge:

W. James, *The Principles of Psychology* (1890), vol. 1, ch. 8, "The Relations of Minds to Other Things," ch. 9, "The Stream of Thought."

G. F. Stout, *Analytic Psychology* (1896), bk. i, ch. 2, "The Analysis of Presentations," ch. 3, "The Apprehension of Form," bk. ii, ch. 5, "Noetic Synthesis."

J. Ward, *Psychological Principles* (1918), ch. 4, secs. 1, 2, "The Presentational Continuum," ch. 6, secs. 1, 2, 6, 7, "Perception," ch. 12, "Thought and Language."

Chapter II, Logical Form

On the structure of complexes:

B. Russell, *Monist*, vol. 28 (1918), pp. 509 ff., "The Philosophy of Logical Atomism," lect. 2.

L. Wittgenstein, *Tractatus Logico-Philosophicus* (1922), Introduction (by B. Russell), Preface, and Text through prop. 4.04.

On identity and diversity:

G. E. Moore, *Proceedings of the Aristotelian Society*, vol. 1 (1900–1901), pp. 103 ff., "Identity."

Chapter III, Universals and Individuals: Order

On universals and individuals:

J. Laird, *A Study in Realism* (1920), ch. 6, "Principles."

S. Alexander, *Space, Time, and Deity* (1920), vol. 1, bk. i, ch. 3, "Universal, Particular, Individual," ch. 4, "Relation."

B. Russell, *Proceedings of the Aristotelian Society*, vol. 12 (1911–1912), pp. 1 ff., "On the Relations of Universals and Particulars."

G. F. Stout, *Proceedings of the British Academy* (1921), "The Nature of Universals and Propositions."

W. E. Johnson, *Logic* (1921), Part I, ch. 11, "The Determinable," ch. 12, "The Relation of Identity," ch. 13, "Relations or Transitive Adjectives."

On space, time, and objects:

C. D. Broad, *Scientific Thought* (1923), ch. 1, "The Traditional Conception of Space and the Principle of Extensive Abstraction," ch. 2, "Time and Change."

A. N. Whitehead, *The Concept of Nature* (1920), ch. 3, "Time," ch. 5, "Space and Motion," ch. 7, "Objects"; also, *The Principles of Natural Knowledge* (1919), Part II, ch. 6, "Events," ch. 7, "Objects."

On abstraction:

G. Berkeley, *The Principles of Human Knowledge* (1710), Introduction.

A. N. Whitehead, *The Principles of Natural Knowledge* (1919), Part III, chs. 8, 9, "Extensive Abstraction."

On order:

B. Russell, *The Principles of Mathematics* (1903), ch. 9, "Relations."

S. Alexander, *Space, Time, and Deity* (1920), vol. i, bk. i, ch. 5, "Order."

Chapter IV, Description and Analysis

On naming, description, and the variable:

B. Russell, *The Problems of Philosophy*, ch. 5, "Knowledge by Acquaintance and Knowledge by Description," ch. 9, "The World of Universals"; also, *The Principles of Mathematics* (1903), ch. 5, "On Denoting," ch. 8, "The Variable"; and *Mind*, N. S., vol. 15 (1905), pp. 479 ff., "On Denoting."

B. Bosanquet, *Logic* (1888), vol. 1, Introduction.

A. N. Whitehead, *An Introduction to Mathematics* (1911), ch. 2, "Variables," ch. 5, "The Symbolism of Mathematics."

On internal and external relations:

F. H. Bradley, *Appearance and Reality* (1893), ch. 2, "Substantive and Adjective," ch. 3, "Relation and Quality."

G. E. Moore, *Philosophical Studies* (1922), ch. 9, "External and Internal Relations."

Chapter V, Truth and Falsity

On truth and falsity:

L. A. Reid, *Knowledge and Truth* (1923), omitting ch. 9 (for a general survey of contemporary theories of truth).

S. Alexander, *Space, Time, and Deity* (1920), vol. 2, bk. iii, ch. 8, "Illusion and Ideas," ch. 9, sec. B, "Truth and Error."

F. H. Bradley, *Essays on Truth and Reality* (1914), ch. 5, "On Truth and Copying," ch. 9, "On Appearance, Error, and Contradiction."

H. H. Joachim, *The Nature of Truth* (1906), ch. 1, "Truth as Correspondence," ch. 3, Part I, "The Coherence Notion of Truth."

On sensation and perception:

Plato, *Theaetetus*, Jowett's translation.

J. Laird, *A Study in Realism* (1920), ch. 2, "The Things We Perceive."

R. F. A. Hoernlé, *Studies in Contemporary Metaphysics* (1920), ch. 4, "On 'Doubting the Reality of the World of Sense'," ch. 5, "'Saving the Appearances' in the Physical World."

W. James, *The Principles of Psychology* (1890), vol. 2, ch. 21, "The Perception of Reality."

B. Russell, *Our Knowledge of the External World* (1914), Lect. 3, "The External World."

C. D. Broad, *Scientific Thought* (1923), ch. 7, "Matter and its Appearances," ch. 8, "The Theory of Sensa."

On the theory of "objectives":

G. D. Hicks, *Mind*, N. S., vol. 31 (1922), pp. 1 ff., "The Philosophical Researches of Meinong."

B. Russell, *Mind*, N. S., vol. 13 (1904), pp. 204 ff., 336 ff., 509 ff., "Meinong's Theory of Complexes and Assumptions."

On belief:

D. Hume, *A Treatise of Human Nature* (1738), bk. i, Part III, secs. 7, 8, 10.

F. C. S. Schiller, *Problems of Belief* (1924), chs. 1, 3, 9, 10, 11, 12.

C. S. Peirce, *Chance, Love, and Logic* (1923), First Paper, "The Fixation of Belief."

Chapter VI, Negation and Contradiction

On negation:

R. Demos, *Mind*, N. S., vol. 24 (1917), pp. 188 ff., "A Discussion of a Certain Type of Negative Proposition."

F. H. Bradley, *The Principles of Logic* (2d ed. 1922), vol. 1, bk. i, ch. 3, "The Negative Judgment," and Terminal Essay 6.

B. Bosanquet, *Logic* (1888), vol. 1, ch. 7, secs. 1, 2, 3, 5.

W. E. Johnson, *Logic* (1921), Part I, ch. 5, "Negation."

H. Bergson, *Creative Evolution* (1911, transl. by A. Mitchell), ch. 4, "The Idea of Nothing."

On contradiction and the "laws of thought":

F. C. S. Schiller, *Formal Logic* (1912), ch. 10, "The Laws of Thought."

C. Sigwart, *Logic* (2d ed. 1895, transl.), vol. 1, ch. 4, "The Negation."

Chapter VII, Formal Deduction

On formal deduction in general:

J. W. Young, *Fundamental Concepts of Algebra and Geometry* (1911), lects. 1, 4, 5, 19, 21.

A. N. Whitehead, *A Treatise on Universal Algebra* (1898), ch. 1, "On the Nature of a Calculus."

C. I. Lewis, *A Survey of Symbolic Logic* (1918), ch. 6, secs. 1, 3.

L. Couturat, *La Logique de Leibniz* (Paris, 1901), ch. 4, sec. 4 ff., "La Caractéristique Universelle."

On the Boolean Algebra:

E. V. Huntington, *Transactions of the American Mathematical Society*, vol. 5, no. 3 (1904), pp. 288 ff., "Sets of Independent Postulates for the Algebra of Logic."

H. M. Sheffer, *Transactions of the American Mathematical Society*, vol. 14, no. 4 (1913), pp. 481 ff., "A Set of Five Independent Postulates for Boolean Algebra."

Chapter VIII, The Metaphysics of Knowledge

On appearance and reality:

F. H. Bradley, *Appearance and Reality* (1893), bk. xi, chs. 13, 14, "The General Nature of Reality," ch. 15, "Thought and Reality," ch. 27, "Ultimate Doubts."

J. Ward, *Naturalism and Agnosticism* (1899), vol. i, Introduction, vol. 2, Part IV, lects. 14, 15.

A. J. Balfour, *A Defence of Philosophic Doubt* (1879), ch. 1, "On the Idea of a Philosophy," ch. 6, "Transcendentalism," ch. 13, "Evolution of Belief."

On the relation of mind and its objects:

W. James, *Essays in Radical Empiricism* (1912), Essay 1, "Does Consciousness Exist?"

S. Alexander, *Space, Time, and Deity* (1920), vol. 2, bk. iii, ch. 4, "Mind and Knowing."

R. B. Perry, *Present Philosophical Tendencies* (1912), Part V, ch. 13, "A Realistic Theory of Knowledge."

On idealism and its critics:

R. F. A. Hoernlé, *Idealism* (1924), especially chs. 4, 5, "Idealism as the Theory of the Absolute."

J. Royce, *The World and the Individual* (1900), vol. 1, lects. 3, 4, 5, 6, 7, 8, 10 (an examination of realism, mysticism, Kantianism, and absolute idealism).

T. H. Greene, *Prolegomena to Ethics* (1883), bk. i, ch. 1, "The Spiritual Principle in Knowledge and in Nature," ch. 2, "The Relation of Man as Intelligence to the Spiritual Principle in Nature."

R. B. Perry, *Present Philosophical Tendencies* (1912), Part III, ch. 6, "The Cardinal Principle of Idealism," ch. 7, "Objective or Transcendental Idealism."

On intuitionism:

H. Bergson, *Creative Evolution* (1911, transl. by A. Mitchell), Introduction, ch. 4, pp. 298 ff., "Form and Becoming"; also *An Introduction to Metaphysics* (1912, trans. by E. Hulme).

General reference:

C. D. Macintosh, *The Problem of Knowledge* (1915).

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